Degrees-of-Freedom of the MIMO Three-Way Channel with Node-Intermittency

Joachim Neu[1], Anas Chaaban[2], Aydin Sezgin[3], Mohamed-Slim Alouini[4]

joachim.neu@tum.de, anas.chaaban@ubc.ca, aydin.sezgin@rub.de, slim.alouini@kaust.edu.sa

Abstract

Recent trends in wireless communications motivate the model of a multiple-input multiple-output (MIMO) three-way channel (3WC) with an intermittent node. We study its degrees-of-freedom (DoF) region and sum-DoF. We devise a non-adaptive encoding scheme and show its DoF region (and thus sum-DoF) optimality for non-intermittent 3WCs and its sum-DoF optimality for node-intermittent 3WCs. However, we show by counterexample that adaptive encoding is necessary to achieve the DoF region of node-intermittent 3WCs.

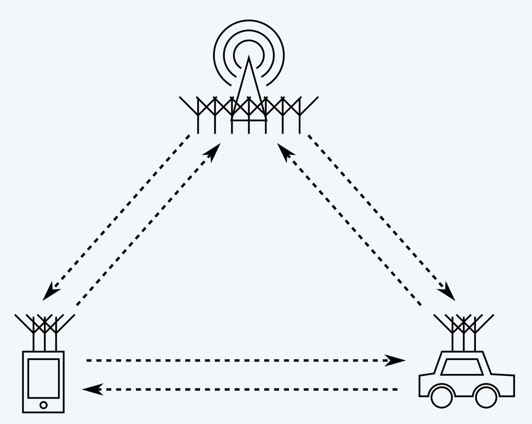
Introduction

Trends in Wireless

- Higher frequencies (*mmWave*)
  → Valuable but fragile line-of-sight
- Local communication (*IoT*, *D2D*, *caching*)
- Multiple antennas (*MIMO*)
- Interference-limited systems

Implications for Channel Models

→ Intermittency as channel impairment
→ Degrees-of-freedom (DoF) perspective
→ Multi-way/device-to-device scenarios



(D2D-scenario with 1 BS and 2 UEs)

Research Questions

- DoF region $\mathscr{D}$?
- Sum-DoF d_{sum}?
- Necessity of adaptive encoding?

Contributions — [1] [2]

- Sum-DoF of node-int. MIMO 3WC: $d_{\text{sum}}^{\text{NI}} = 2\bar{\tau}\min\{M_2, M_3\} + 2\tau\,\text{mid}\{M_1, M_2, M_3\}$
  → Non-adaptive encoding suffices
  $\text{mid}\{\} \triangleq$ "middle value"
- Necessity of adaptive encoding for DoF region of node-int. MIMO 3WC $\mathscr{D}^{\text{NI}}$:
  - Converses on $\mathscr{D}^{\text{NI}}$ (adaptive) and $\mathscr{D}^{\text{NI}}_{\text{!adapt}}$ (non-adaptive)
  - Adaptive encoding scheme that achieves a $\boldsymbol{d} \notin \mathscr{D}^{\text{NI}}_{\text{!adapt}}$, ergo $\mathscr{D}^{\text{NI}} \setminus \mathscr{D}^{\text{NI}}_{\text{!adapt}} \neq \emptyset$

  → Adaptive encoding is necessary
- DoF region $\mathscr{D}^{\emptyset\text{I}}$ of non-int. MIMO 3WC, $M_1 \geq M_2 \geq M_3$ (wlog):

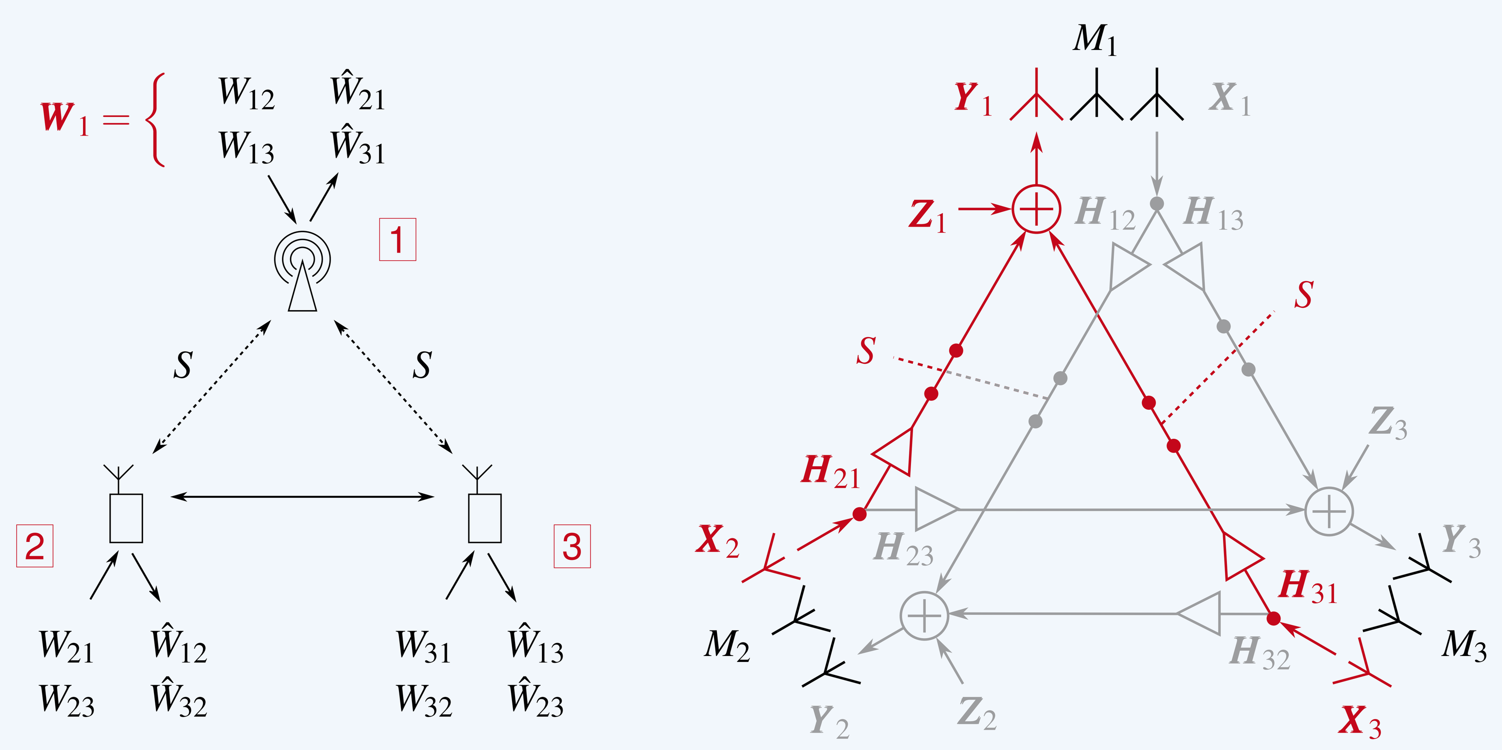
$$\max\{d_{12}+d_{13}+d_{23}, d_{12}+d_{13}+d_{32}\} \leq M_1 \qquad \max\{d_{21}+d_{13}+d_{23}, d_{12}+d_{31}+d_{32}\} \leq M_2$$

$$\max\{d_{21}+d_{31}+d_{32}, d_{21}+d_{31}+d_{23}\} \leq M_1 \qquad \max\{d_{31}+d_{32}, d_{13}+d_{23}\} \leq M_3$$

$$\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \geq 0$$

→ Non-adaptive encoding suffices

System Model



∀ time instances ℓ: $\boldsymbol{Y}_{1,\ell} = S_\ell \boldsymbol{H}_{21}\boldsymbol{X}_{2,\ell} + S_\ell \boldsymbol{H}_{31}\boldsymbol{X}_{3,\ell} + \boldsymbol{Z}_{1,\ell}$ (analogous for $\boldsymbol{Y}_{2,\ell}$ and $\boldsymbol{Y}_{3,\ell}$)

- Numbers of RX/TX antennas: M_1, M_2, M_3
- Intermittency state sequence S^n known causally: $S_\ell \sim \text{Bern}(\tau), \bar{\tau} \triangleq 1-\tau$
- Gains $\boldsymbol{H}_{ij}$ known a priori
- $\boldsymbol{Z}_{i,\ell} \sim \mathscr{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{M_i})$, $\sum_{\ell=1}^n \mathbb{E}[\|\boldsymbol{X}_{i,\ell}\|_2^2] \leq nP$, SNR $\rho \triangleq \frac{P}{\sigma^2}$
- DoFs $d_{ij} \triangleq \limsup_{\rho\to\infty} \frac{R_{ij}(\rho)}{\log(\rho)}$
- Non-adaptive encoding: $\boldsymbol{x}_{i,\ell} = \mathscr{E}_{i,\ell}(\boldsymbol{w}_i)$
- Adaptive encoding: $\boldsymbol{x}_{i,\ell} = \mathscr{E}_{i,\ell}(\boldsymbol{w}_i, \boldsymbol{y}_{i,1}, \ldots, \boldsymbol{y}_{i,\ell-1})$

Key References

[1] J. Neu, A. Chaaban, A. Sezgin, and M.-S. Alouini. "Degrees-of-Freedom of the MIMO Three-Way Channel with Node-Intermittency". arXiv:1708.08161.
[2] A. Chaaban, A. Sezgin, and M.-S. Alouini. "On the Degrees-of-Freedom of the MIMO Three-Way Channel with Intermittent Connectivity". In: *Proc. IEEE Int. Symp. Inf. Theory (ISIT)* (2017).
[3] V. R. Cadambe and S. A. Jafar. "Interference Alignment and Degrees of Freedom of the K-User Interference Channel". In: *IEEE Trans. Inf. Theory* 54.8 (Aug. 2008), pp. 3425–3441.

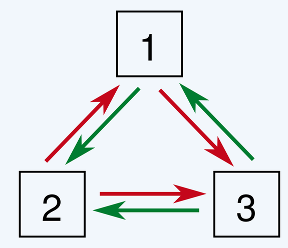
Converses

Example for $M_1 \geq M_2 \geq M_3$

Partition the sum-DoF: $d_{\text{sum}} = d_{13} + d_{23} + d_{21} + d_{12} + d_{32} + d_{31}$

Genie-aided converse bounds:

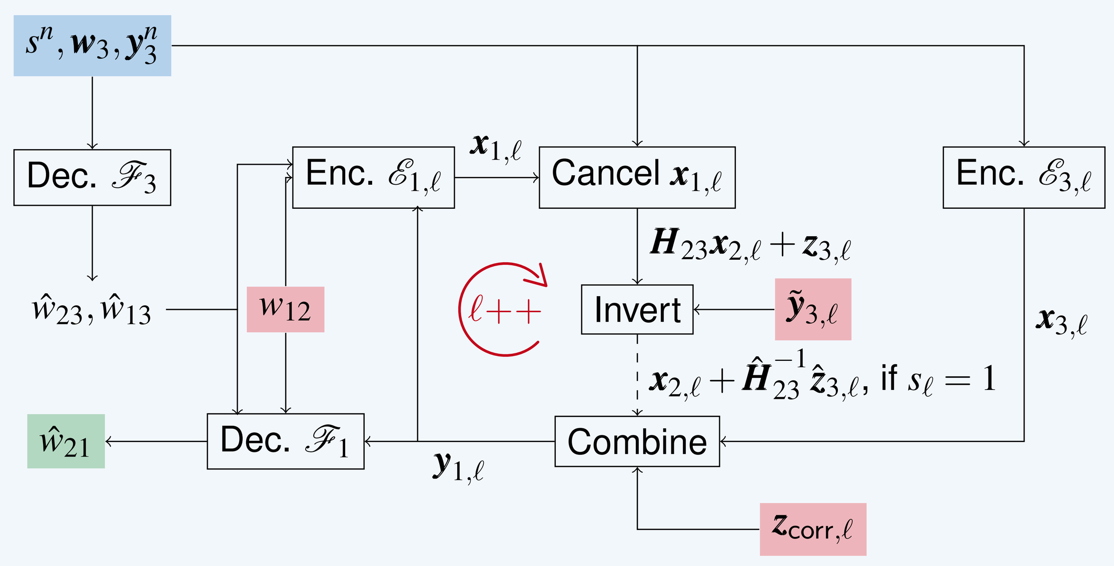
- Enable 3 to act like 1 and decode W_{21}
- Enable 2 to act like 1 and decode W_{31}



Side information required at 3:

- Enable decoding: W_{12}
- Compensate antenna size: $\tilde{\boldsymbol{Y}}_{3,\ell} \triangleq S_\ell(\tilde{\boldsymbol{H}}_{23}\boldsymbol{X}_{2,\ell} + \tilde{\boldsymbol{Z}}_{3,\ell}) \quad\rightarrow\quad$ stack $\boldsymbol{Y}_3^n$ and $\tilde{\boldsymbol{Y}}_3^n \quad\rightarrow\quad \hat{\boldsymbol{Y}}_3^n$
- Noise correction: $\boldsymbol{Z}_{\text{corr},\ell} \triangleq \boldsymbol{Z}_{1,\ell} - S_\ell(\boldsymbol{H}_{21}\hat{\boldsymbol{H}}_{23}^{-1}\hat{\boldsymbol{Z}}_{3,\ell})$
- ◦ Compensate intermittency

Iteratively recover $\boldsymbol{y}_{1,\ell}$ for $\ell = 1, \ldots, n$ using side information w_{12}, $\tilde{\boldsymbol{y}}_3^n$, $\boldsymbol{z}_{\text{corr}}^n$, then decode $\hat{w}_{21}$:



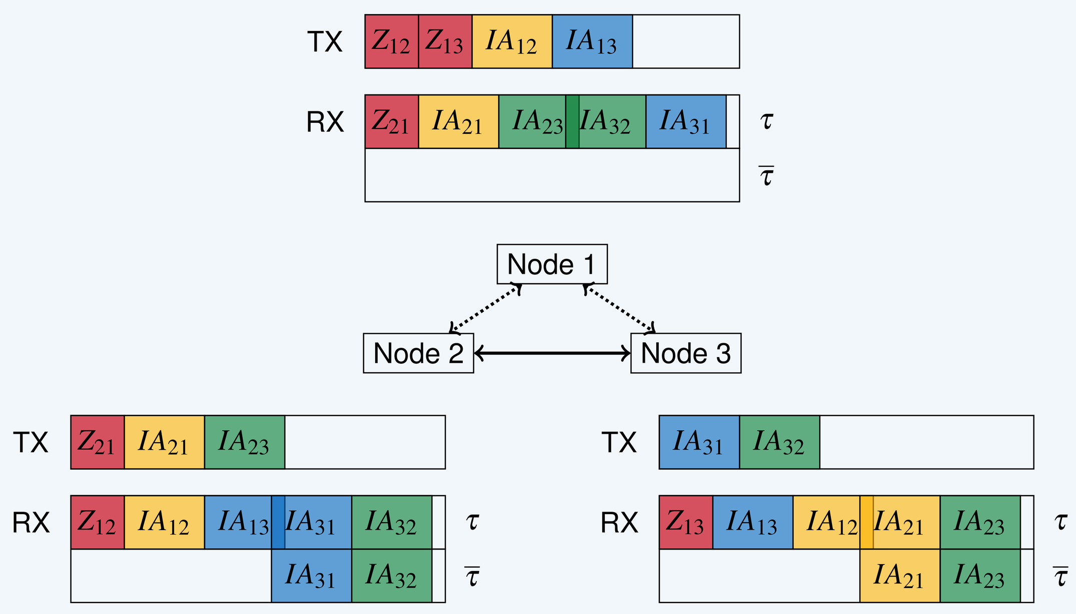
$$n(R_{13}+R_{23}+R_{21}-\varepsilon_n) \leq I(W_{13}W_{23}W_{21}; \boldsymbol{W}_3\boldsymbol{Y}_3^n S^n W_{12}\tilde{\boldsymbol{Y}}_3^n\boldsymbol{Z}_{\text{corr}}^n) \leq \ldots$$

$$= \sum_{\ell=1}^{n} \left[I(\boldsymbol{X}_{1,\ell}\boldsymbol{X}_{2,\ell}; \hat{\boldsymbol{Y}}_{3,\ell} \mid S_\ell) + I(\boldsymbol{Z}_{\text{corr},\ell}; \hat{\boldsymbol{Z}}_{3,\ell} \mid S_\ell)\right] \leq n(\tau M_2 + \bar{\tau} M_3)\log(\rho) + o\{\log(\rho)\}$$

$$d_{13} + d_{23} + d_{21} \leq \tau M_2 + \bar{\tau} M_3$$

Achievability

Example for $M_1 \geq M_2 \geq M_3$

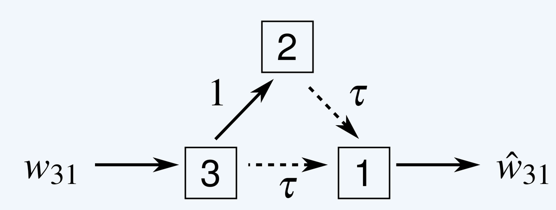


- ZF_{ij}: dim. of $i \to j$ sent using zero-forcing
- IA_{ij}: dim. of $i \to j$ sent using interfer.-alignm. [3]

→ System of inequal. in d_{ij}
→ Achievable $\mathscr{D}_{\text{achiev}}$: Fourier-Motzkin elimination
→ Achievable $d_{\text{sum,achiev}}$: linear programming

DoF Region Counterexample

Let $M_1 \geq M_2 \geq M_3$, $\tau M_2 \geq M_3$:



Converse (non-adaptive encoding):

$$n(R_{31} - \varepsilon_n) \leq I(W_{31}; \boldsymbol{W}_1\boldsymbol{Y}_1^n S^n \boldsymbol{W}_2)$$

$$\ldots \quad \text{(using } \boldsymbol{x}_2^n = \mathscr{E}_2^n(\boldsymbol{w}_2)\text{)}$$

$$\leq n\tau M_3 \log(\rho) + o\{\log(\rho)\}$$

$$d_{31} \leq \tau M_3$$

Achievability (adaptive encoding):

Multi-hop relaying 3 → 2 → 1, backwards decoding/successive interference canc. at 1

$$d_{31} = M_3$$

Conclusion

- The presented non-adaptive encoding scheme ...
  ... achieves $d_{\text{sum}}^{\text{NI}} = d_{\text{sum,achiev}}$ of the node-intermittent channel
  ... achieves $\mathscr{D}^{\emptyset\text{I}} = \mathscr{D}_{\text{achiev}}$ of the non-intermittent channel
  → Non-adaptive encoding suffices for $d_{\text{sum}}^{\text{NI}}$ and $\mathscr{D}^{\emptyset\text{I}}$
  ... constitutes an inner bound $\mathscr{D}_{\text{achiev}} \subset \mathscr{D}^{\text{NI}}$ for the node-intermittent channel
- But any non-adaptive scheme provides only a strict inner bound for $\mathscr{D}^{\text{NI}}$
  → Adaptive encoding is necessary to achieve $\mathscr{D}^{\text{NI}}$

[1] **TUM – Technical University of Munich**
Department of Electrical and Computer Engineering
Institute for Communications Engineering

[2] **UBC – University of British Columbia**
School of Engineering, UBC Okanagan Campus
Communication Theory Lab

[3] **RUB – Ruhr University Bochum**
Department of Electrical Engineering and Information Technology
Chair for Digital Communication Systems

[4] **KAUST – King Abdullah University of Science and Technology**
Computer, Electrical and Mathematical Sciences & Engineering
Communication Theory Lab