Quantized Polar Code Decoders: Analysis and Design

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Thank you: Rüdiger Urbanke[EPFL], Gerhard Kramer[TUM]

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Objective & Motivation

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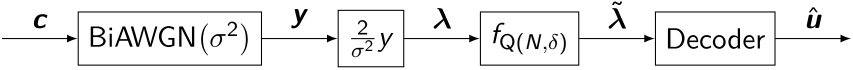
➔ Sizable gains, particularly for low code rates

Preliminaries

System Model



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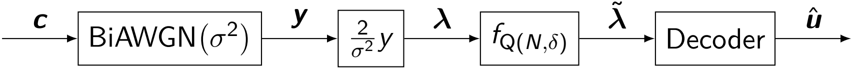

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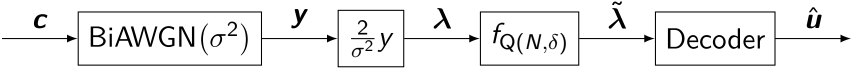

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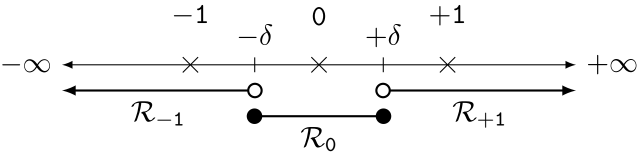

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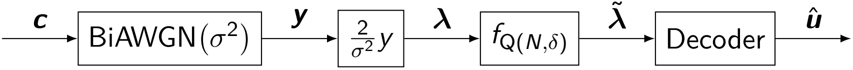

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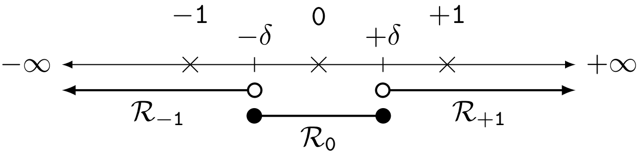

Other quantization: $\mathcal{L}_7 \triangleq \{0, \pm 1, \pm 2, \pm 3\} \subseteq \mathbb{Z}$ $\mathcal{L}_\infty \triangleq \mathbb{R}$

PC Basics & SC Decoding [Stolte 2002] [Arıkan 2009]

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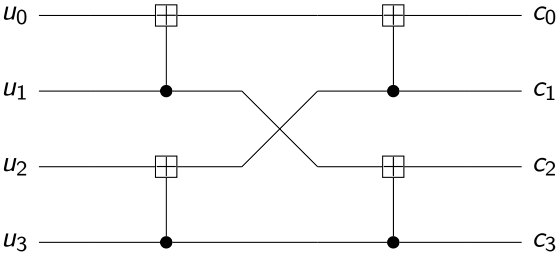

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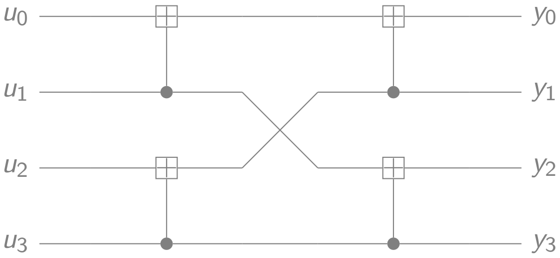

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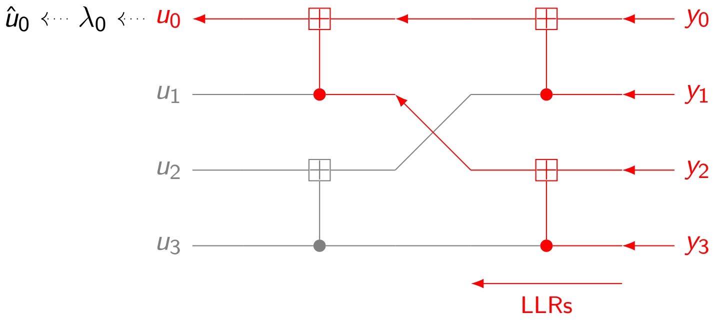

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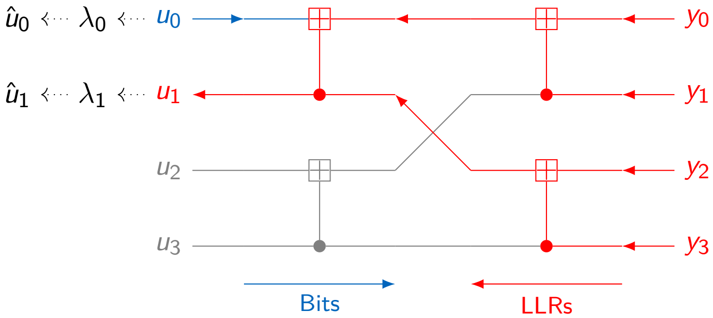

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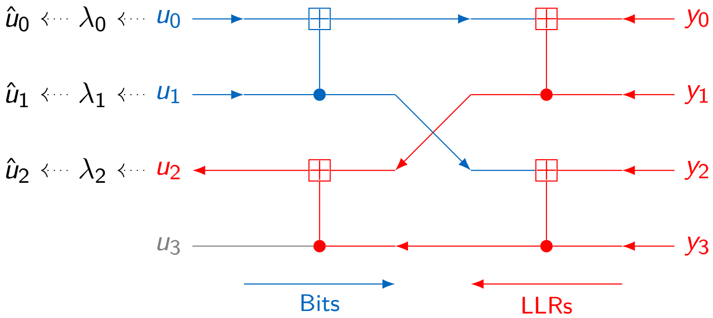

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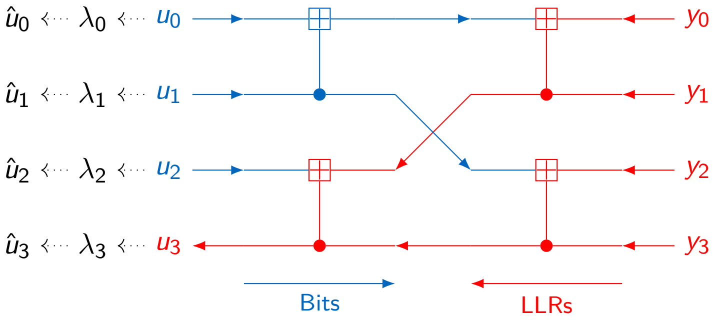

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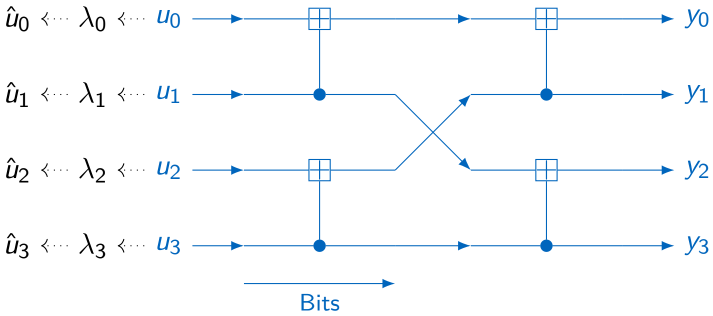

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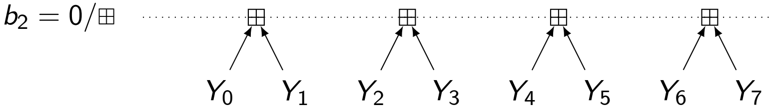

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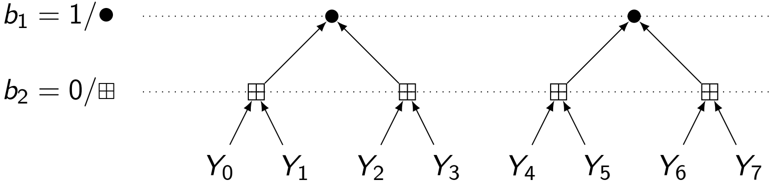

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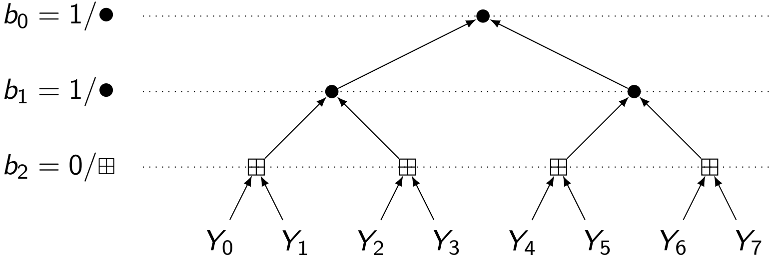

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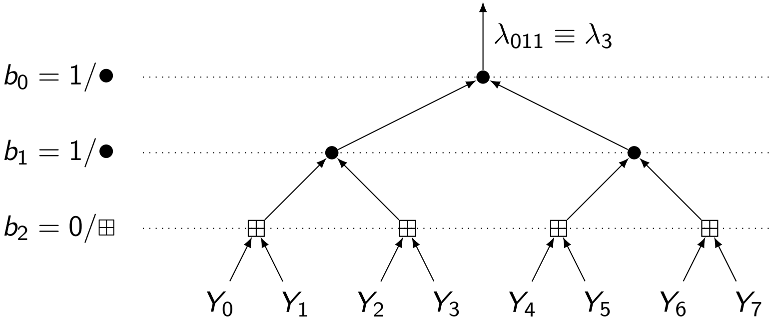

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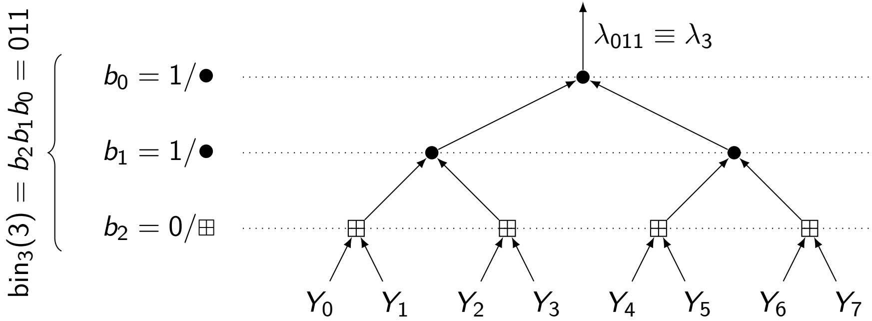

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List size $L = 2$:

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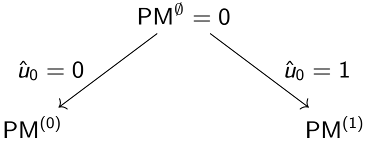

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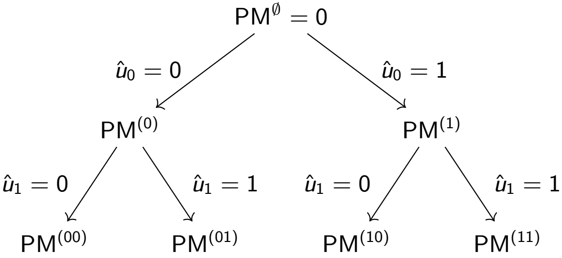

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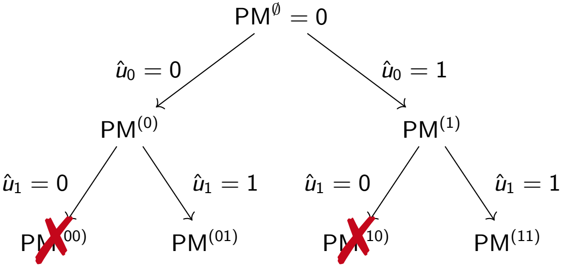

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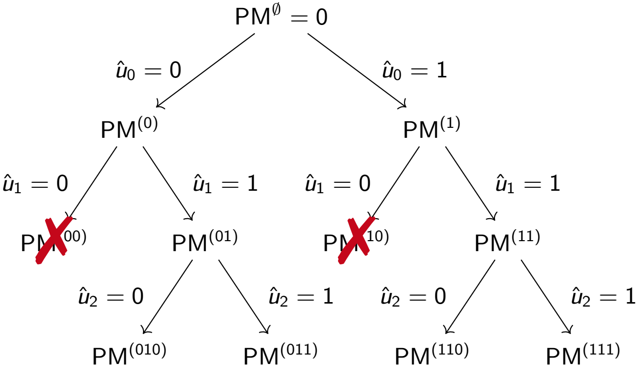

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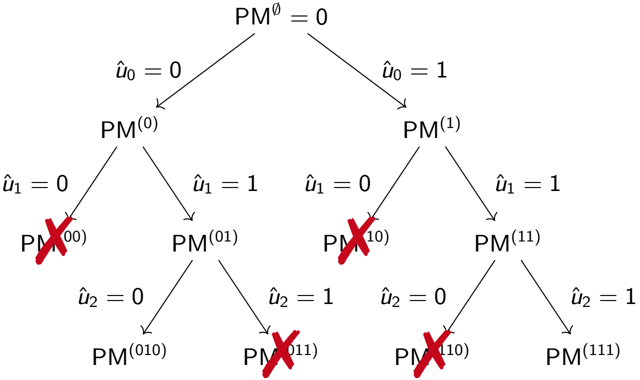

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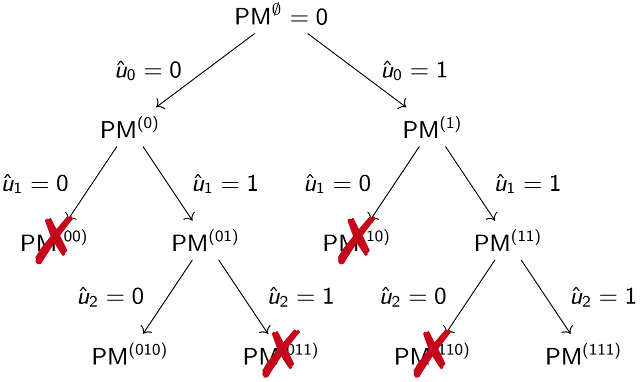
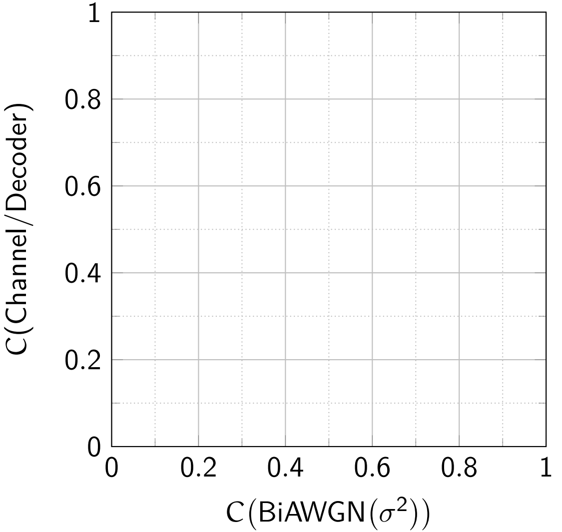


→ List $\left\{(010, \mathrm{PM}^{(010)}), (111, \mathrm{PM}^{(111)})\right\}$ of length $L = 2$

3Q Decoding

3Q-SC Decoding Capacity [Hassani and Urbanke 2012]



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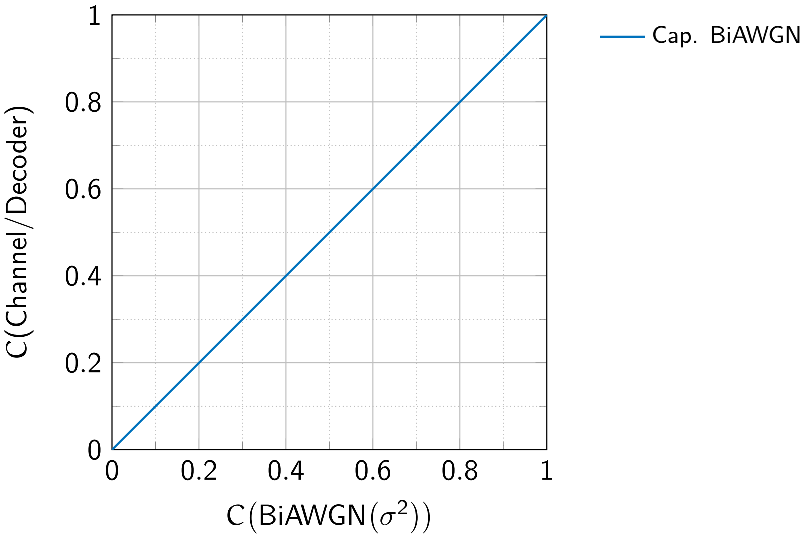

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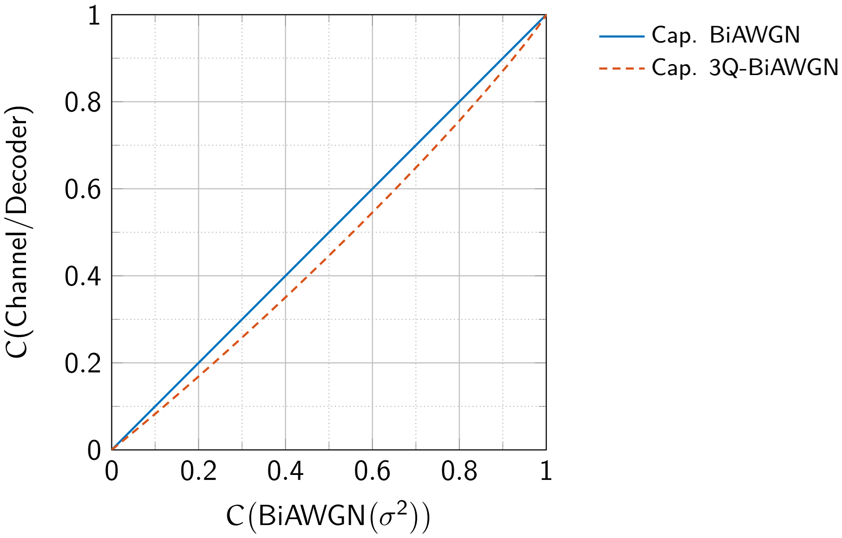

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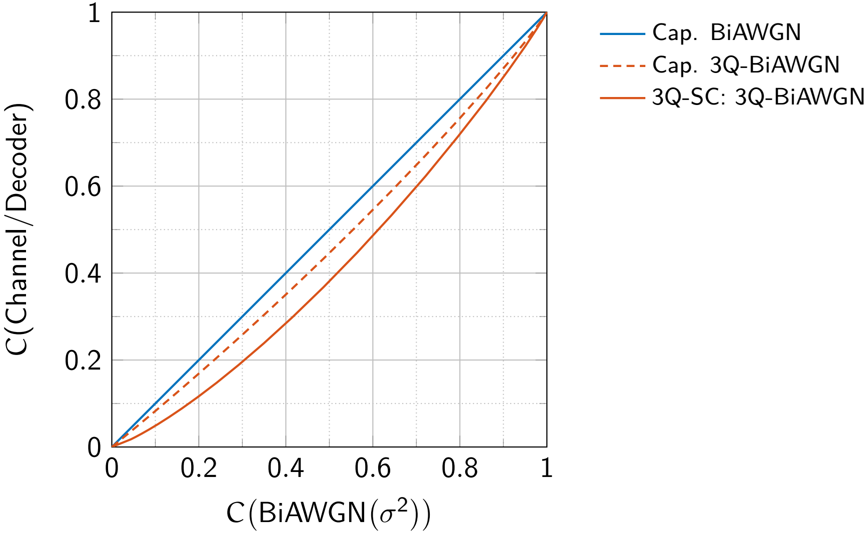

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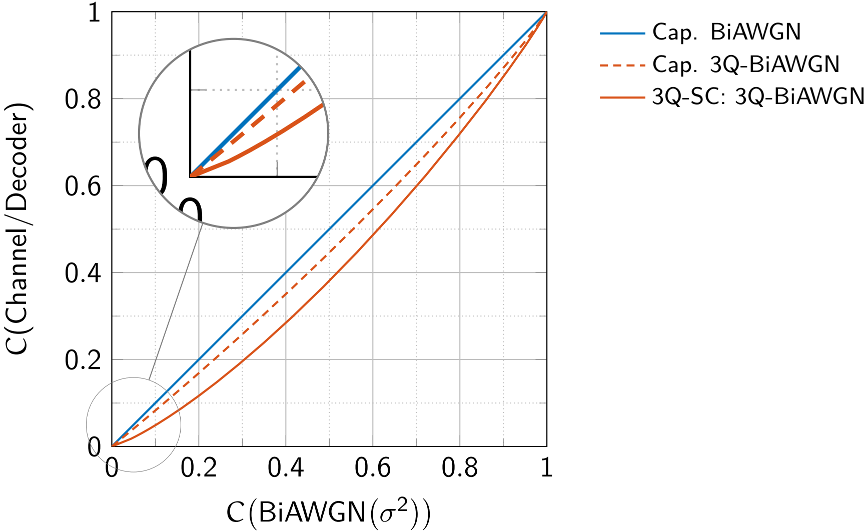

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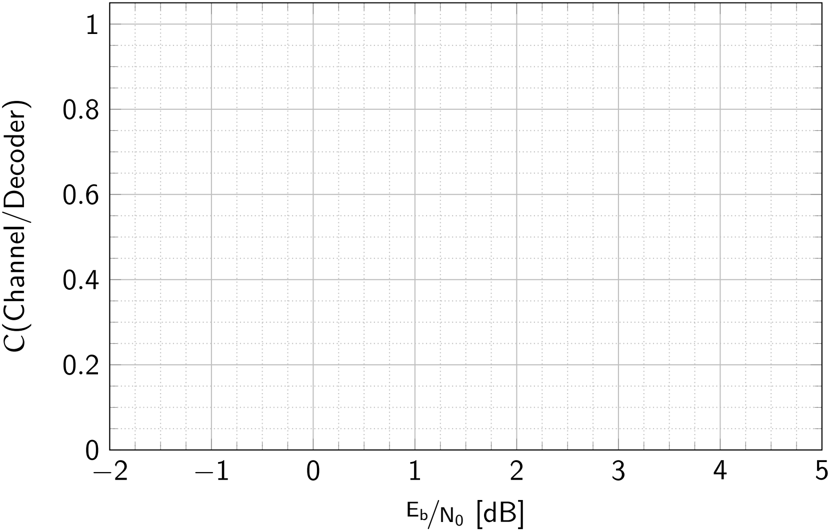

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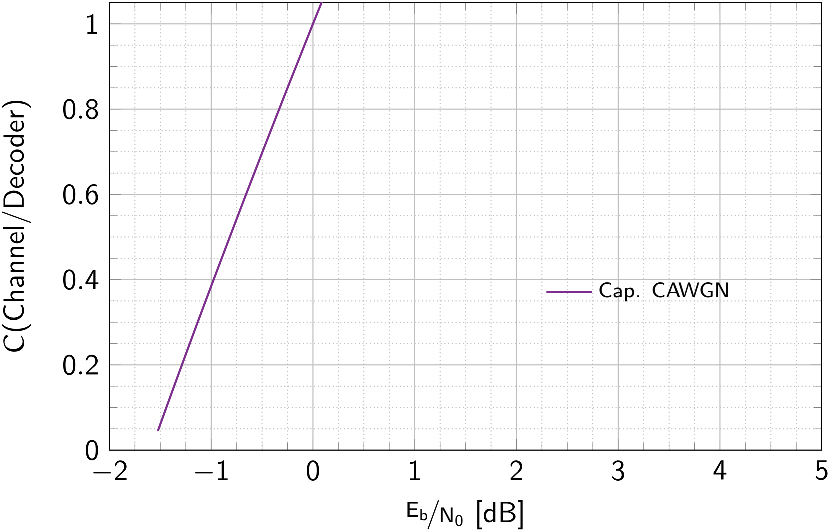

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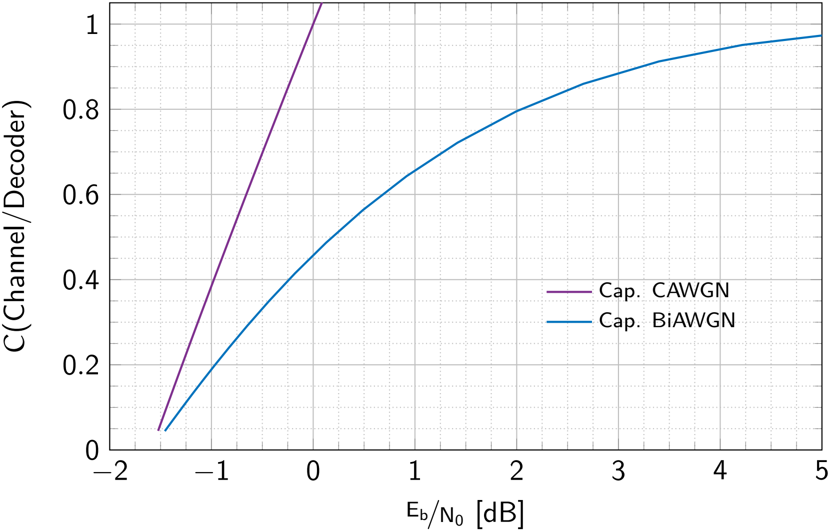

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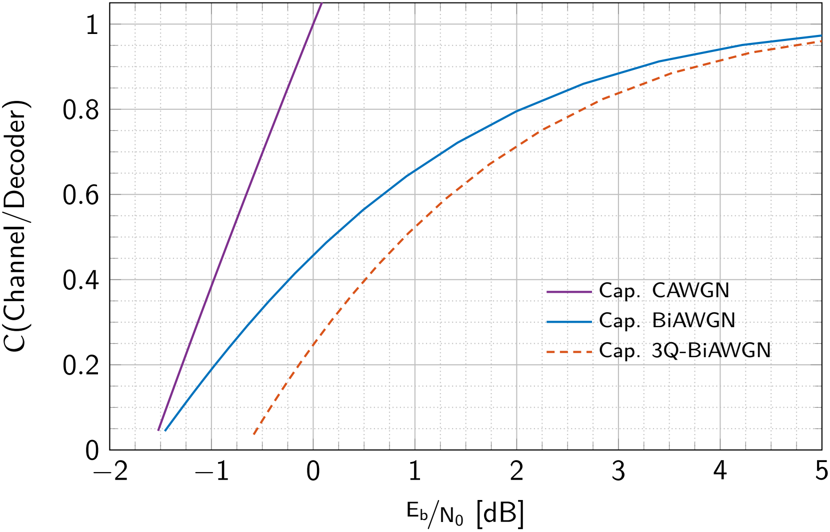

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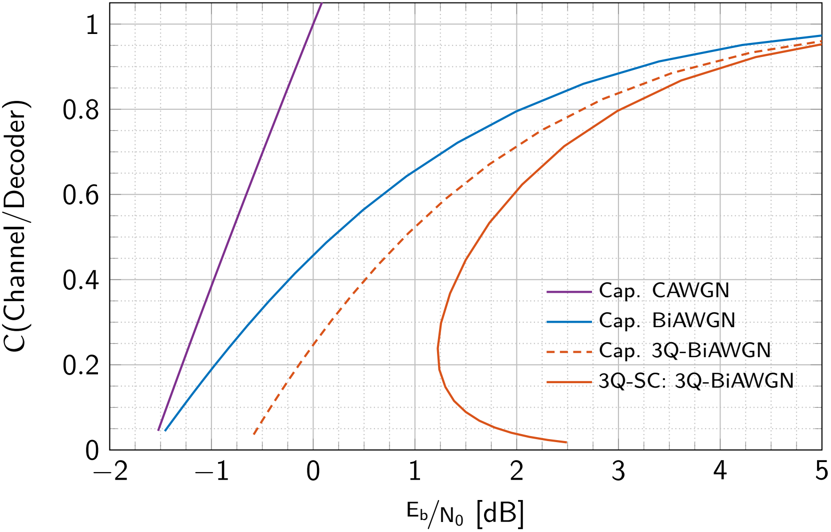

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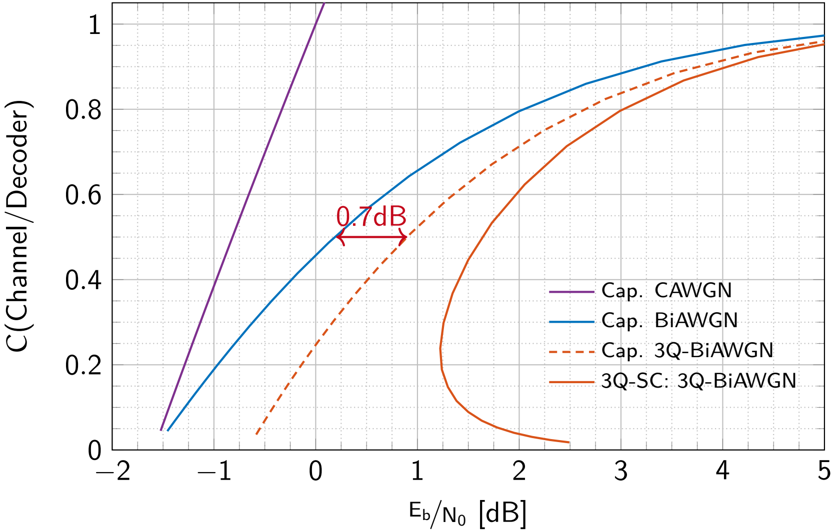

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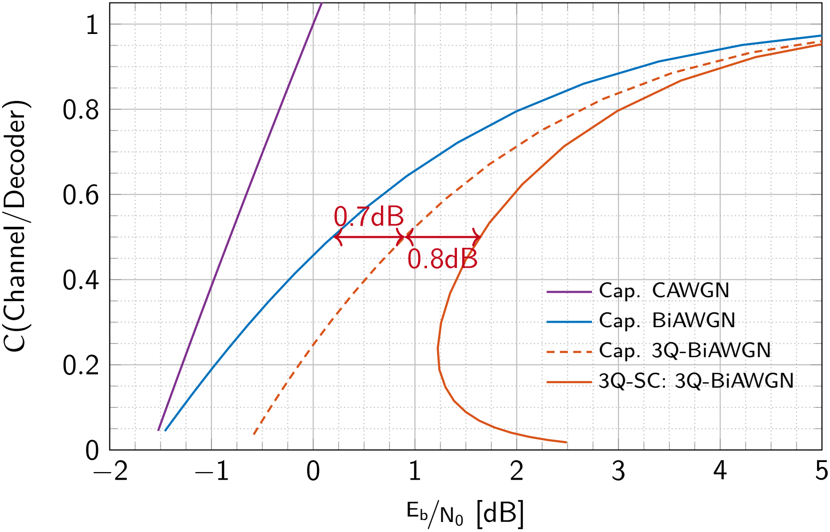

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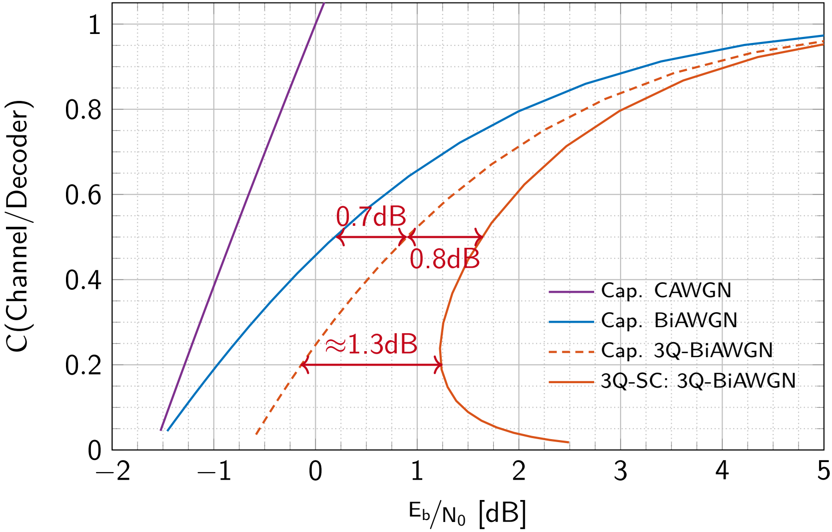

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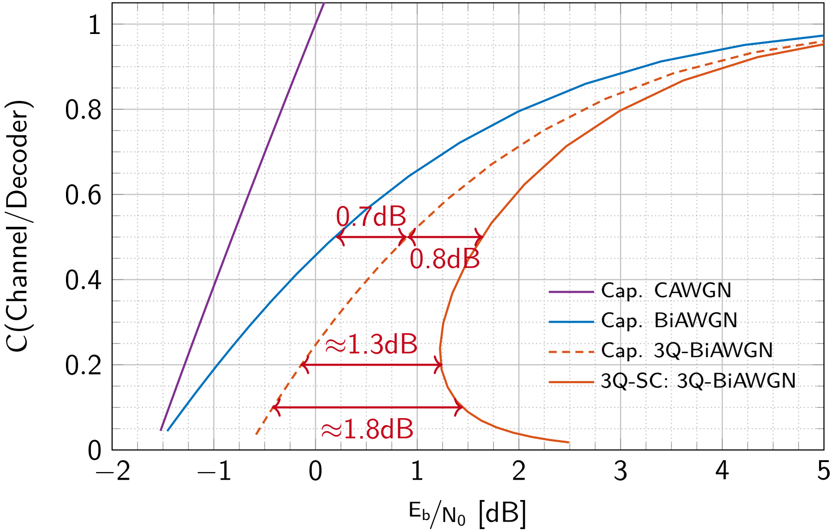

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➔ Use *ML-among-list* to declare winning codeword!

3Q-SCL Decoding: Impact of Coarse Quantization

Unquantized decoder:

- Full-precision LLRs $\lambda \in \mathcal{L}_\infty$
- Full-precision PMs:

$$\mathrm{PM} = -\ln(\Pr[\boldsymbol{U} = \hat{\boldsymbol{u}} | \boldsymbol{Y} = \boldsymbol{y}])$$

$\approx$ Plausibility of path (ML)

Quantized decoder:

- Distorted LLRs $\tilde{\lambda} \in \mathcal{L}_3$

  ➔ No magnitude, no reliability info
- Distorted PMs

  ➔ Bad plausibility measure

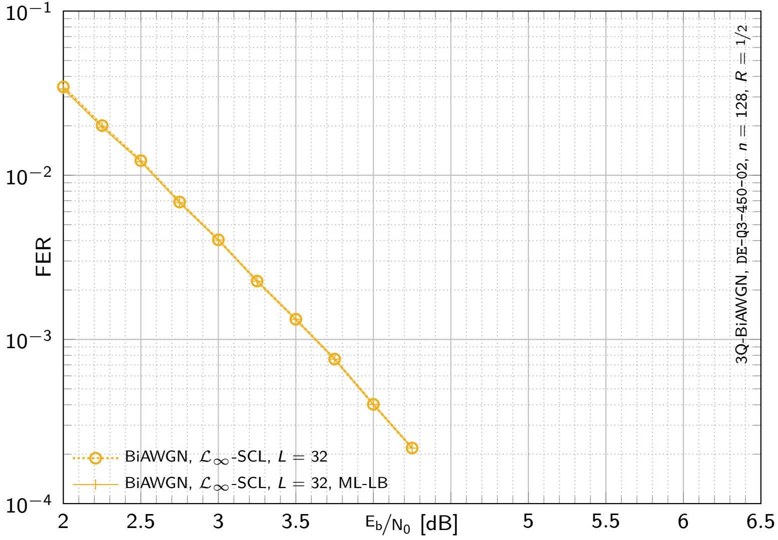
PM quantization severely impacts list management!

➔ Use *ML-among-list* to declare winning codeword!

➔ Use statistical reliability info in *expected path metric updates*!

Mitigation Techniques

Maximum-Likelihood among List: $n = 128$



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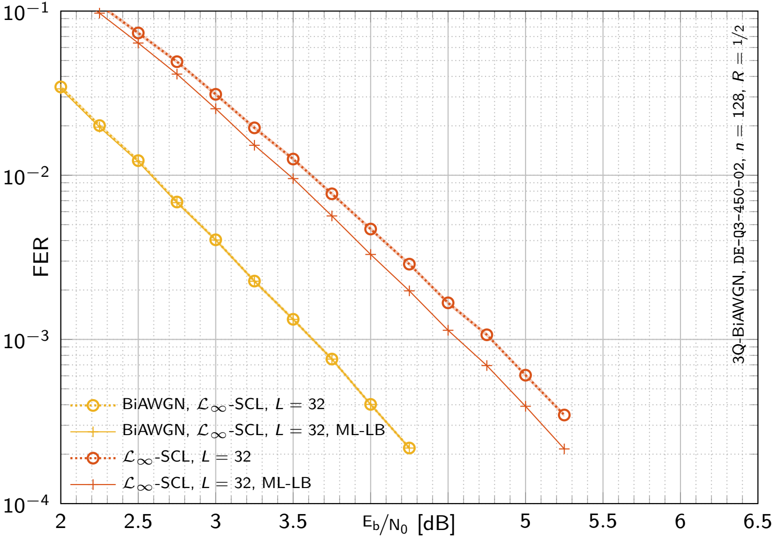

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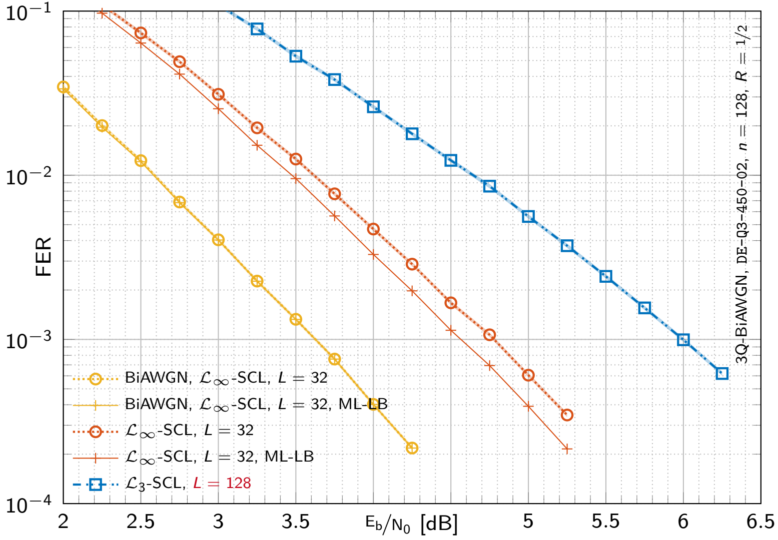

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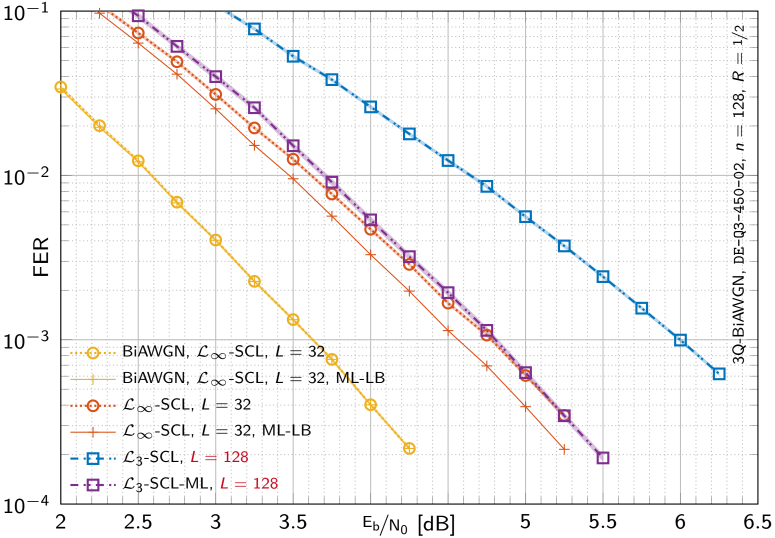
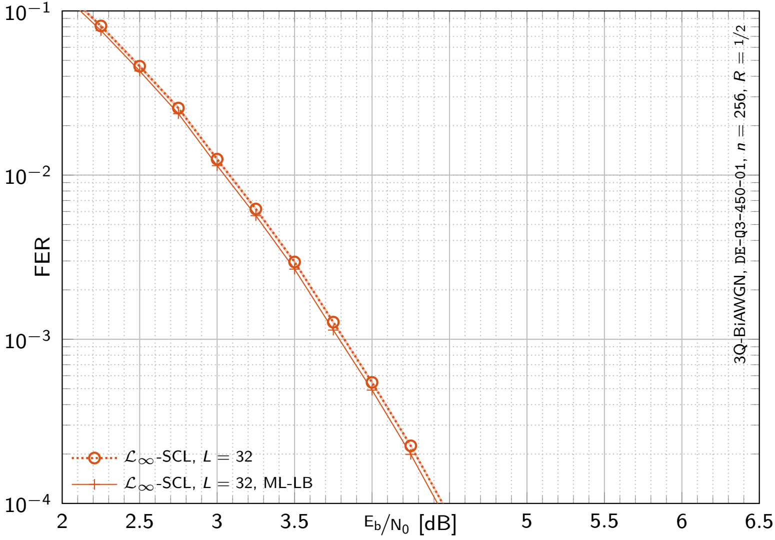


Maximum-Likelihood among List: $n = 256$



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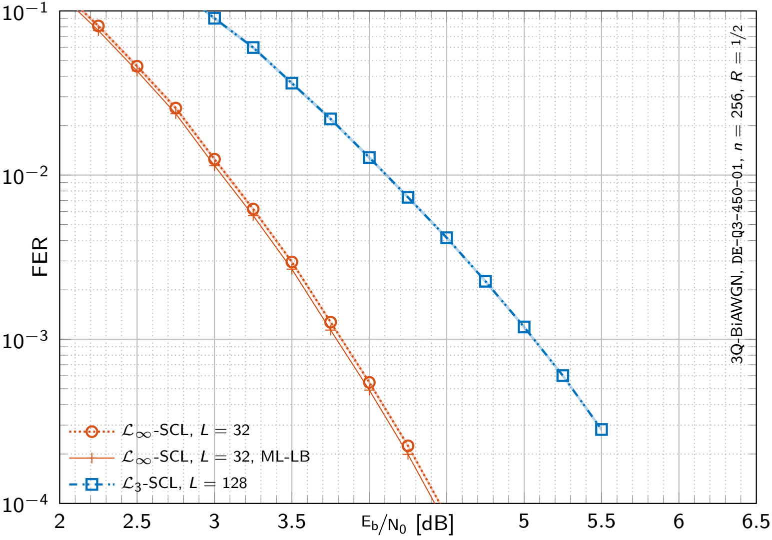

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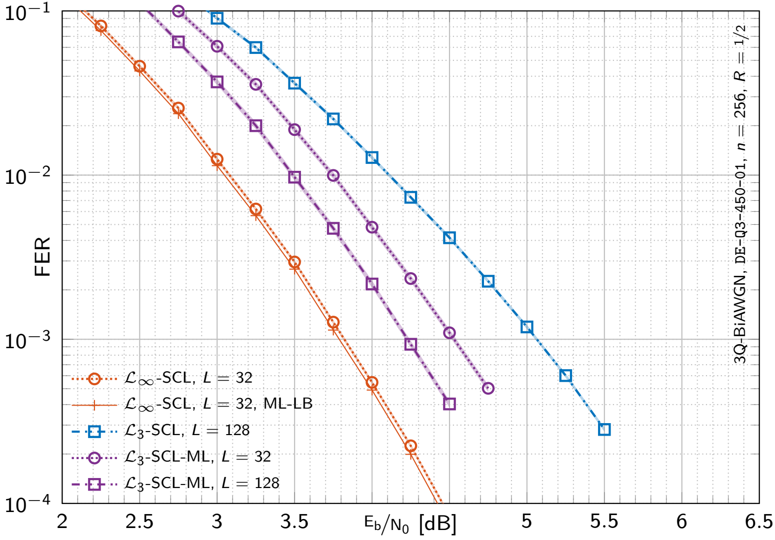

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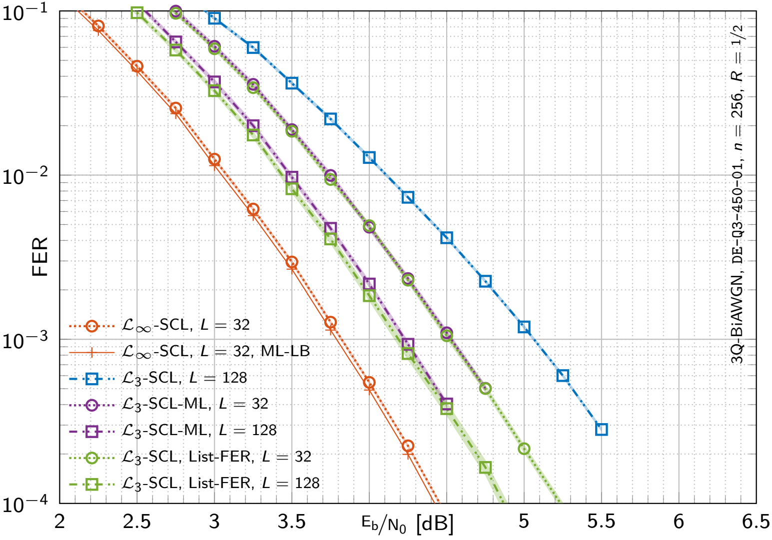

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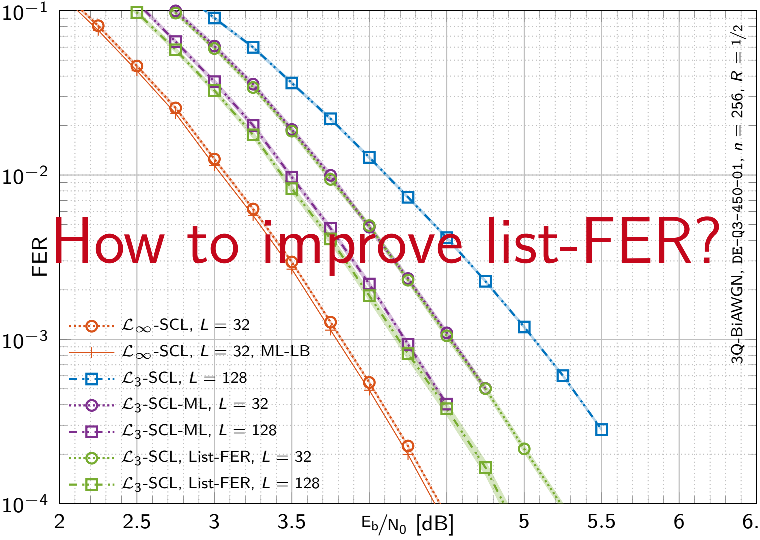

Expected Path Metric Updates: Idea

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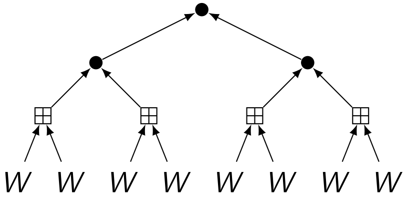
Unquantized decoder:



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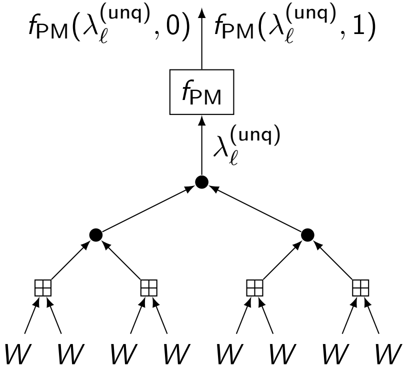

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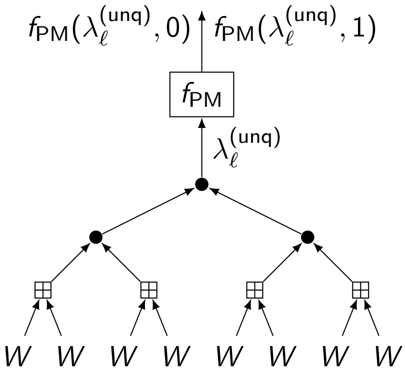

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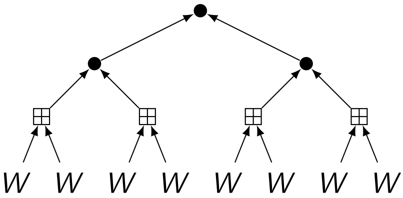

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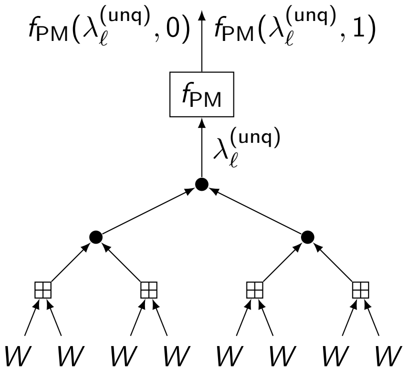

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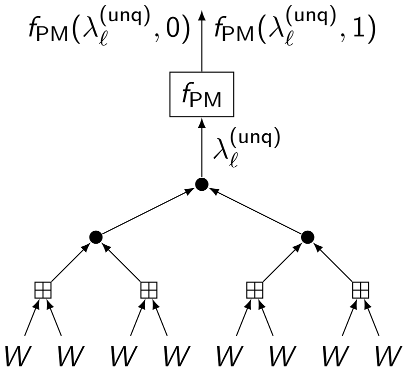

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MMSE: $$\hat{f}_{\mathsf{PM}}(\lambda^{(\mathsf{q})}, j) \triangleq \mathrm{E}\left[f_{\mathsf{PM}}(\Lambda^{(\mathsf{unq})}, j) \middle| \Lambda^{(\mathsf{q})} = \lambda^{(\mathsf{q})}\right]$$

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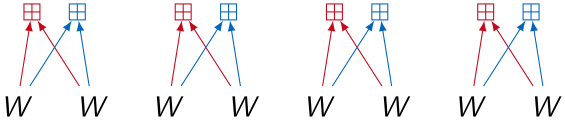
➔ How to obtain $P\left(\Lambda^{(\mathsf{unq})}, \Lambda^{(\mathsf{q})}\right)$?

Expected Path Metric Updates: Density Evolution

$W \quad W \quad W \quad W \quad W \quad W \quad W \quad W$

Expected Path Metric Updates: Density Evolution



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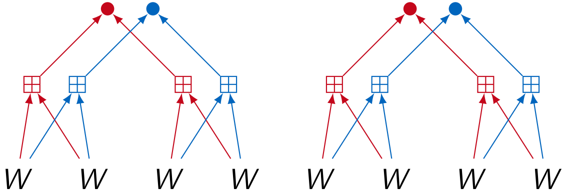

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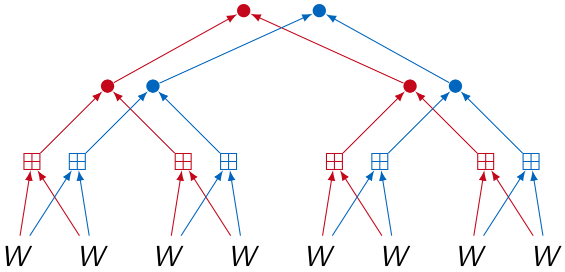

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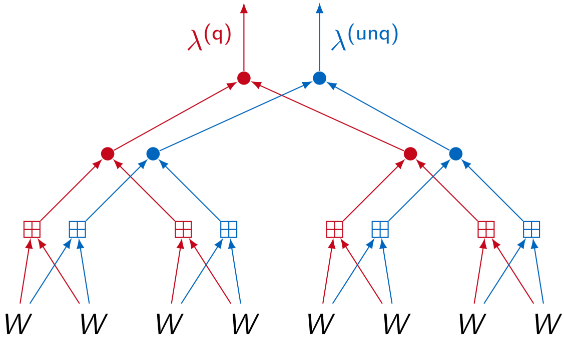

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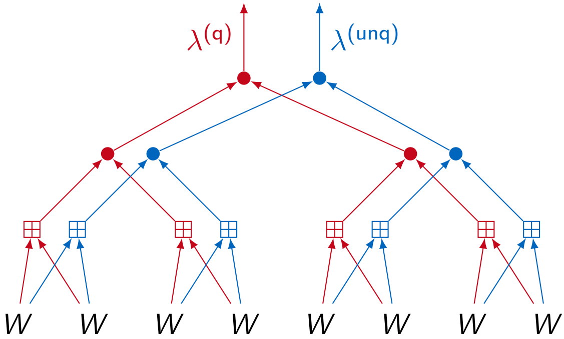

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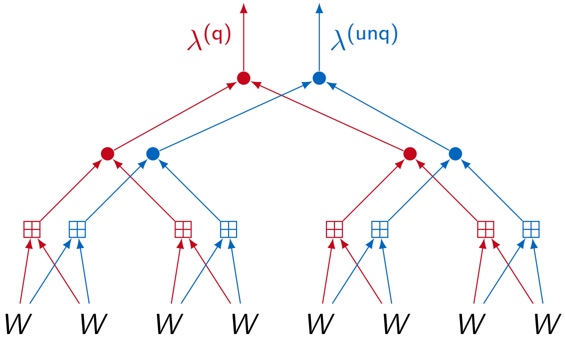

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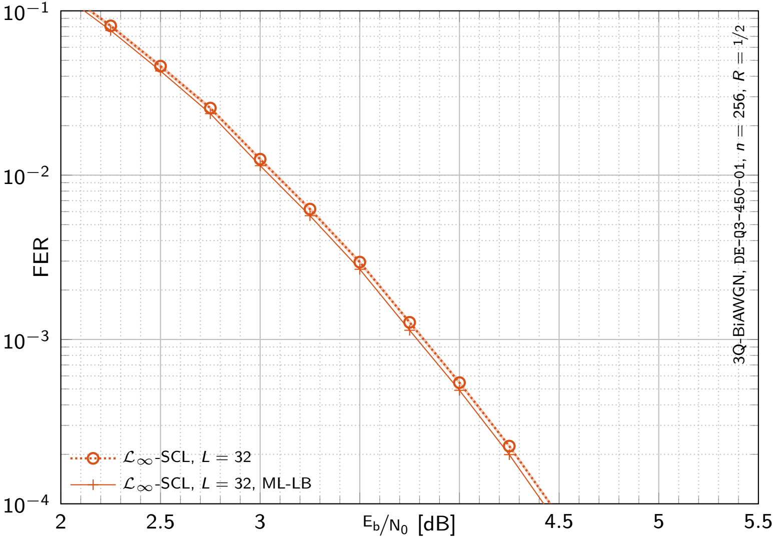

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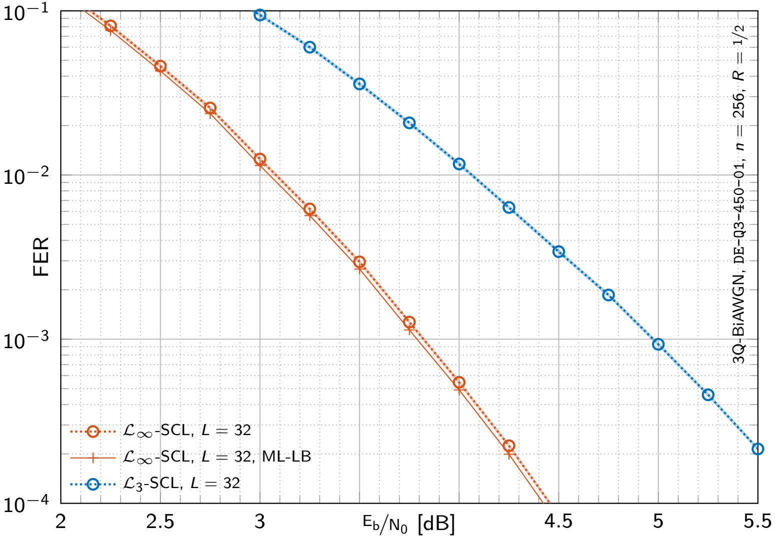

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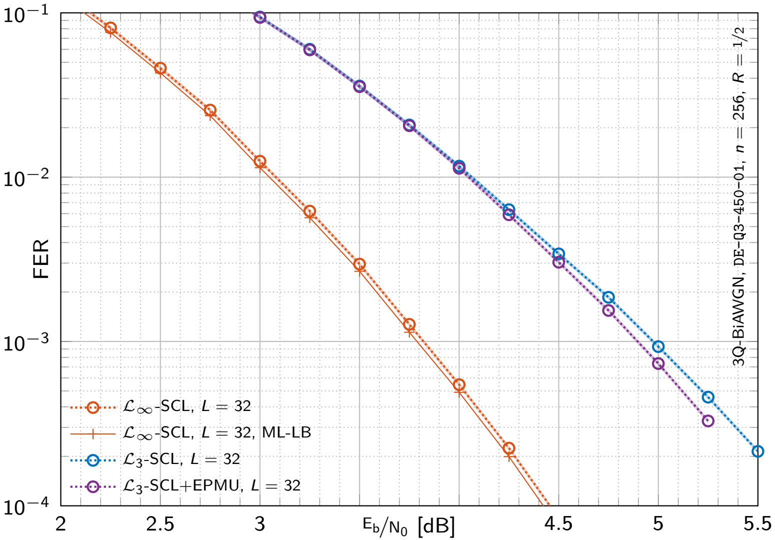

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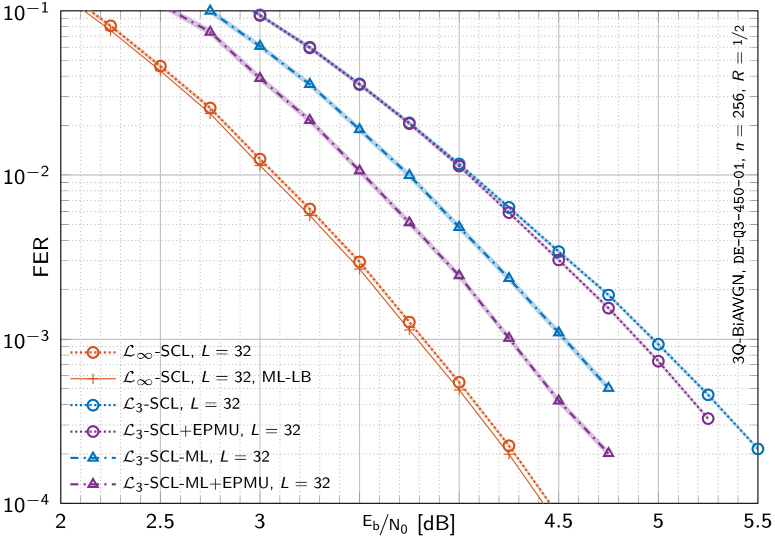

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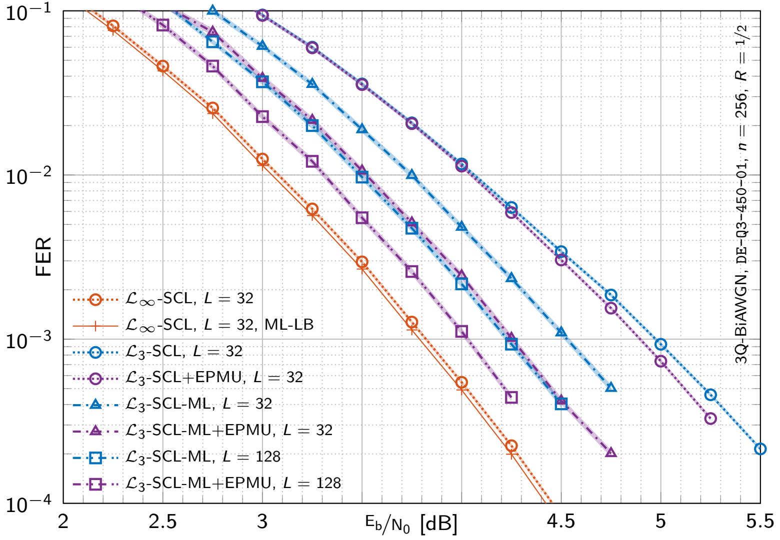

EPMU with Contradiction Counting

Idea: Count contradictions at variable nodes as low-complexity instantaneous reliability indicator!

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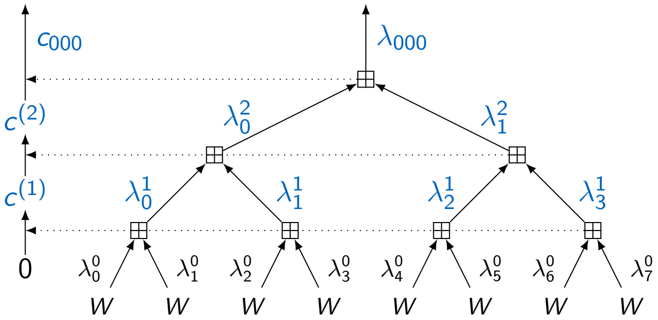

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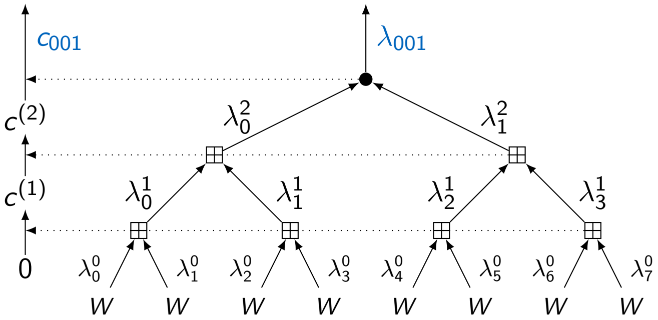

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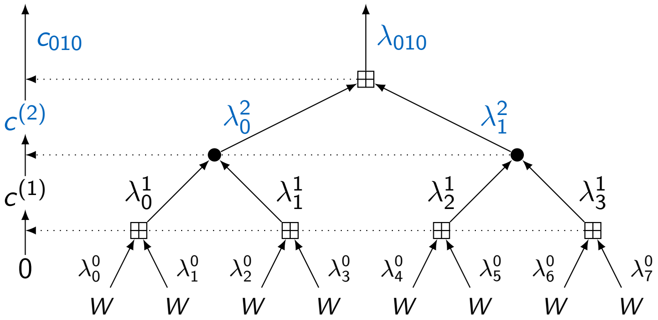

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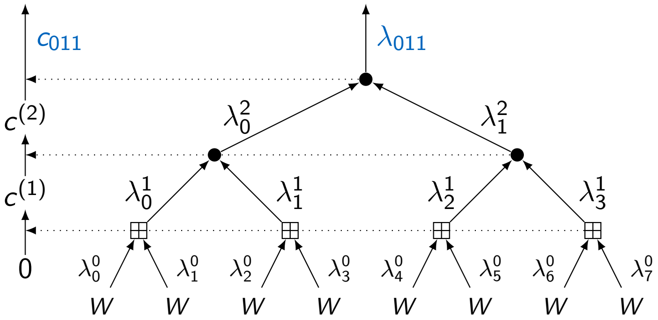

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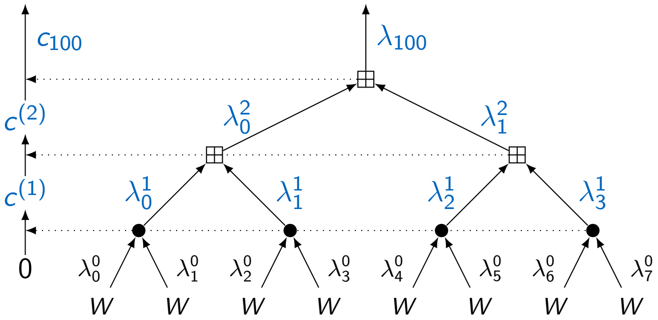

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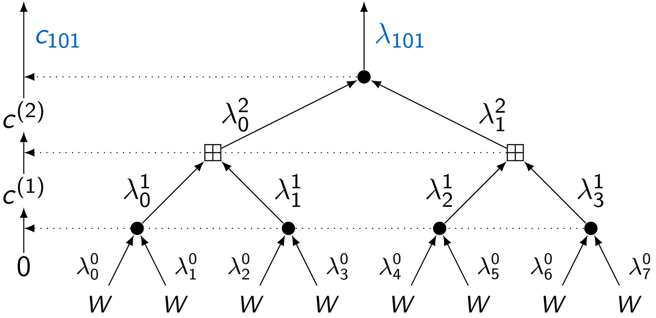

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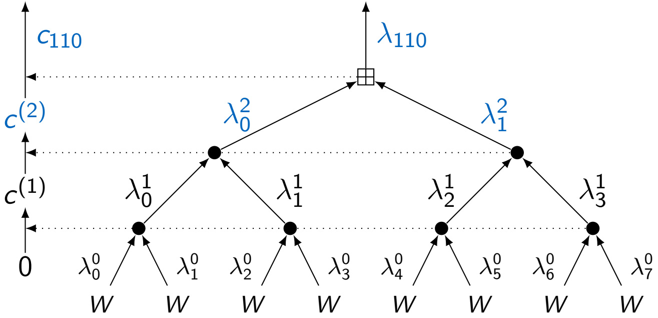

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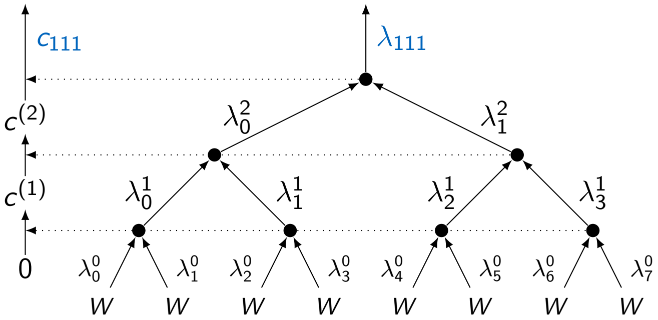
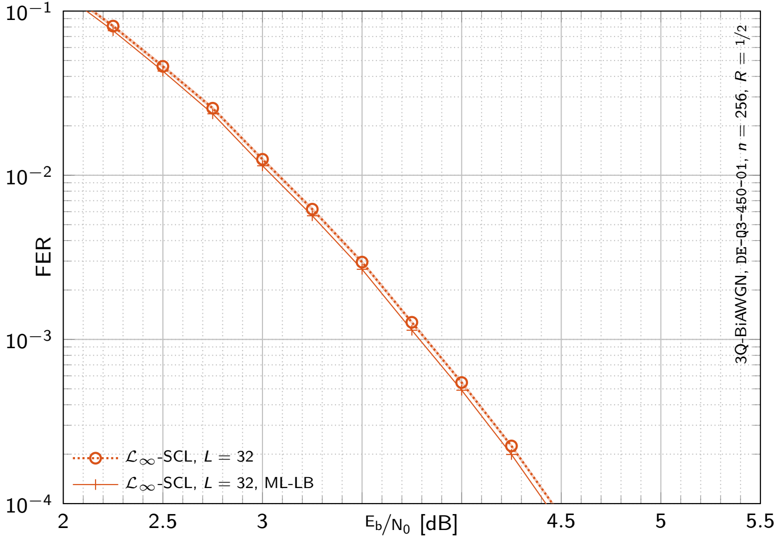


EPMU with Contradiction Counting: Experiments



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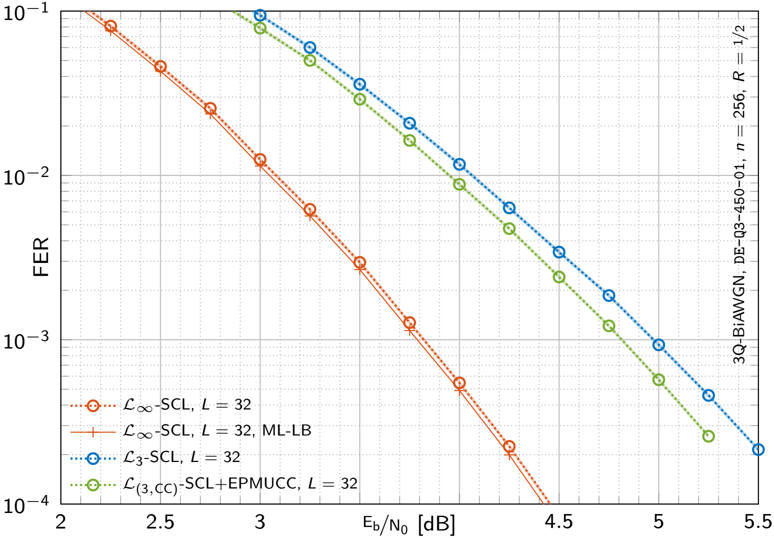

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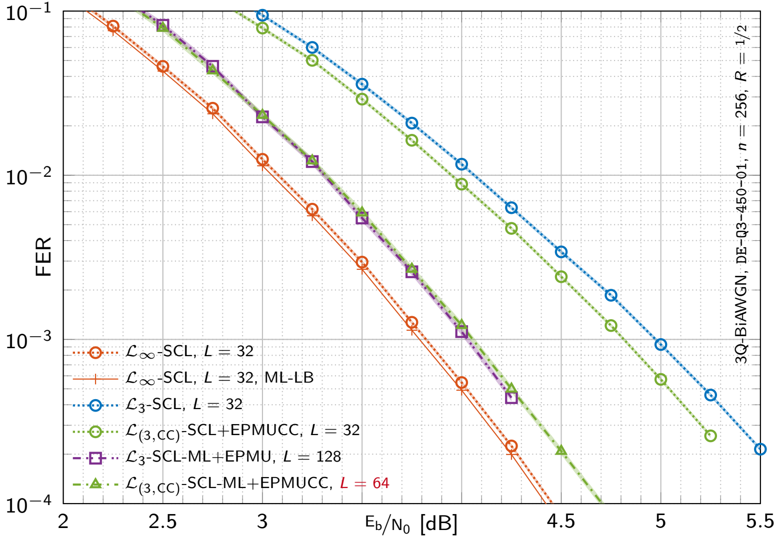
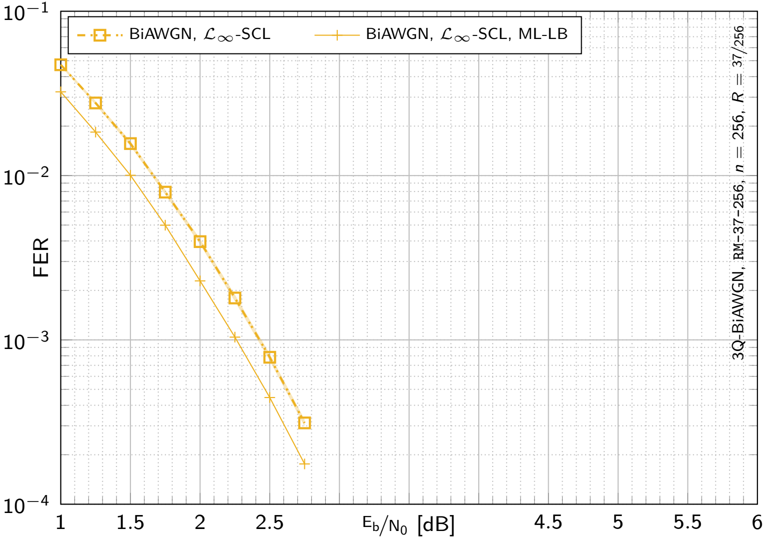


Validation / Robustness

Validation: Low Code Rate: $R \approx 0.145$, $L = 128$



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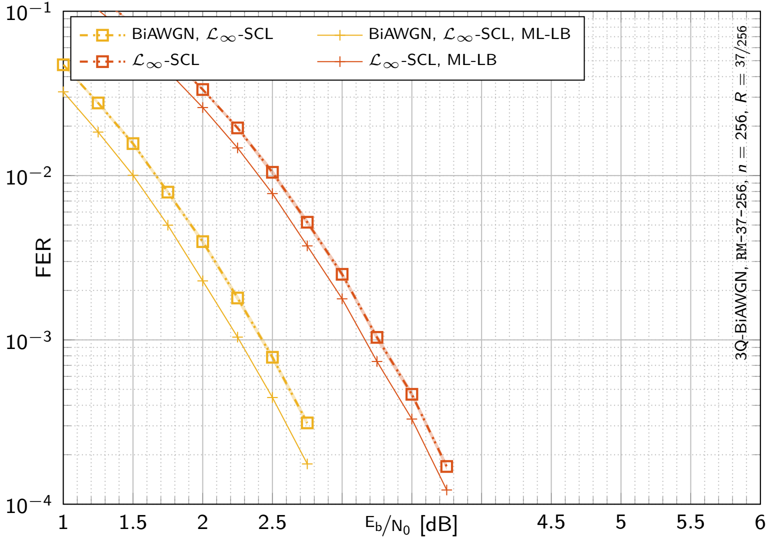

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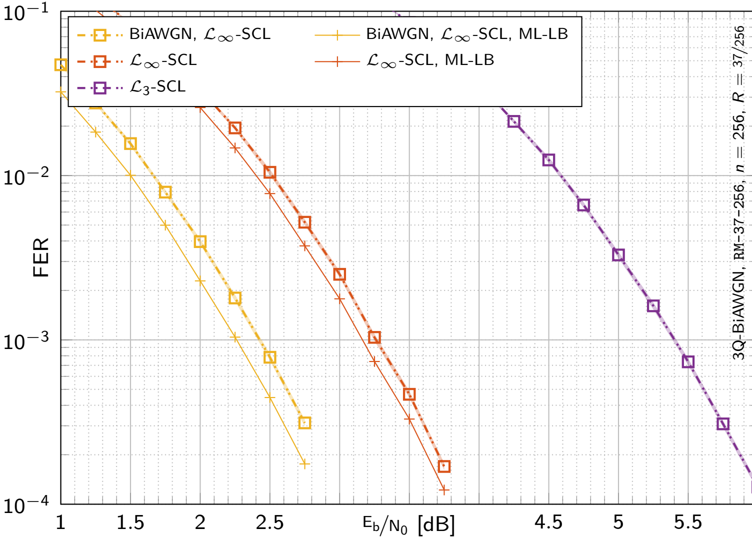

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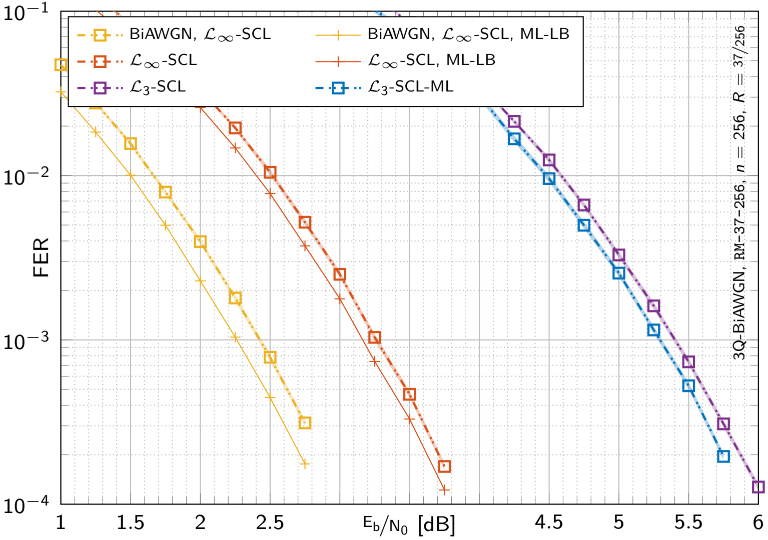

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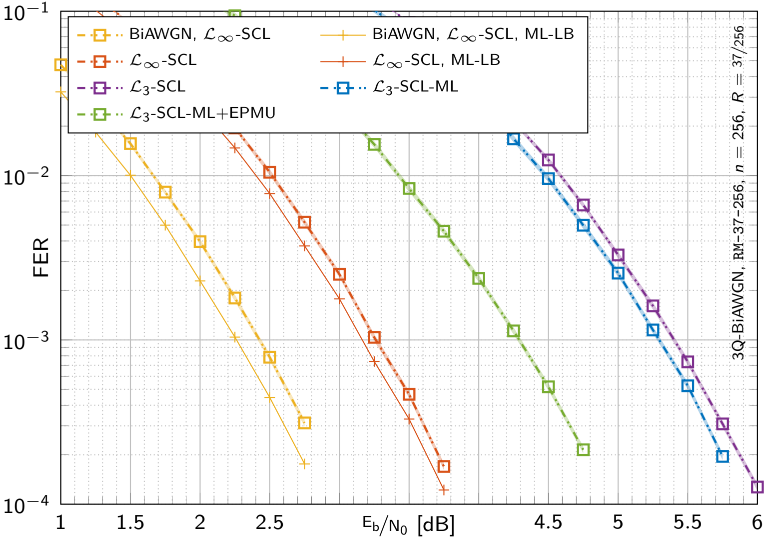
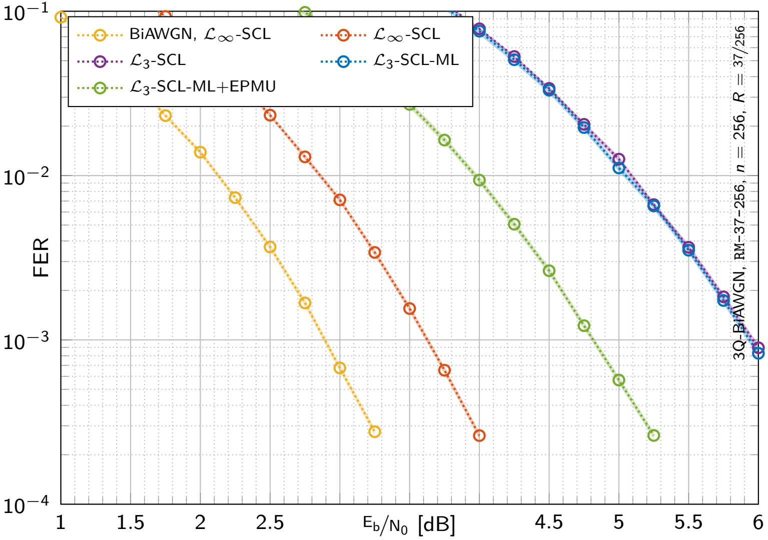
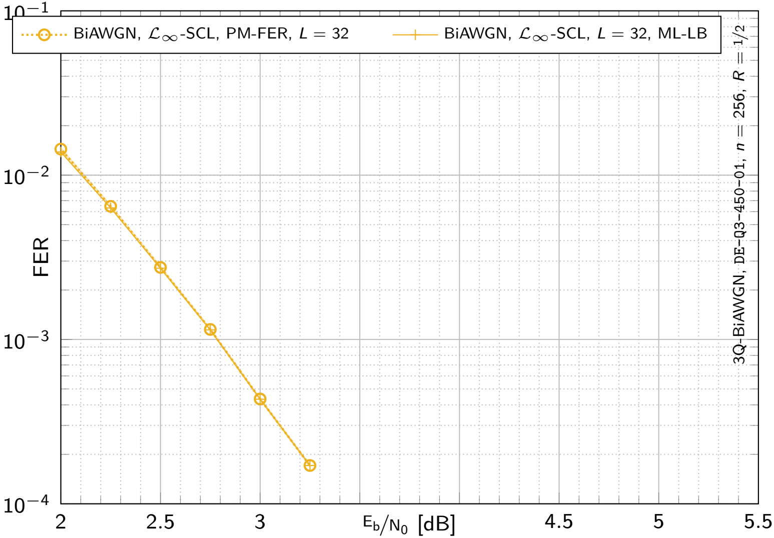


Validation: Low Code Rate: $R \approx 0.145$, $L = 32$



Comparison: Q7 Performance



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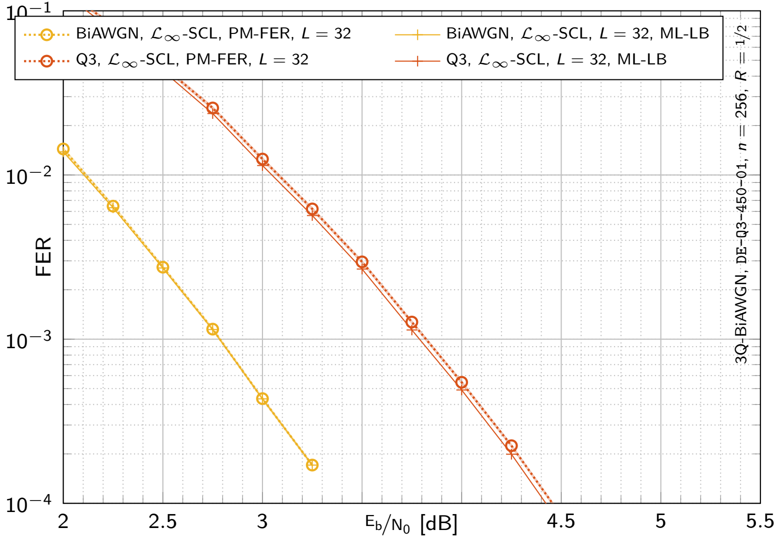

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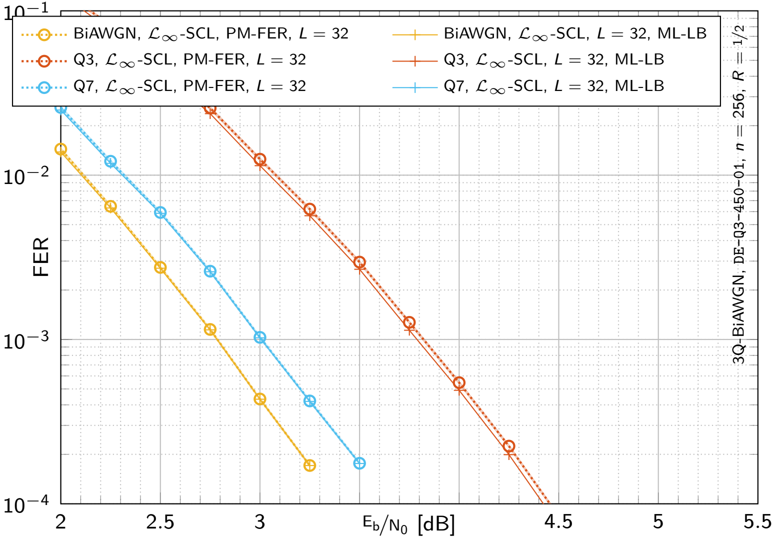

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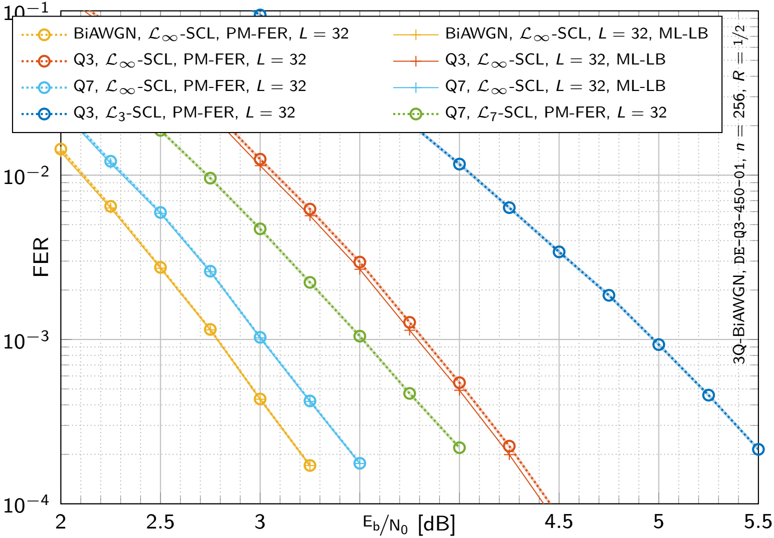

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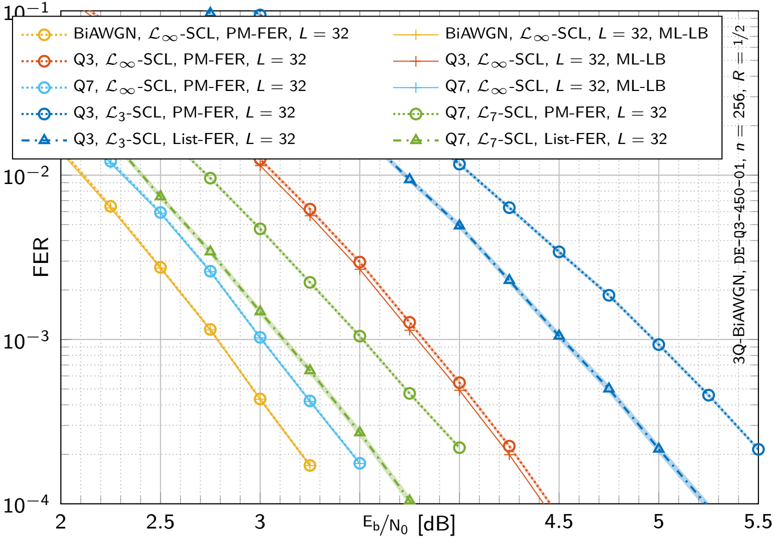

Comparison: ‘1st Layer Unquantized Then Q3’

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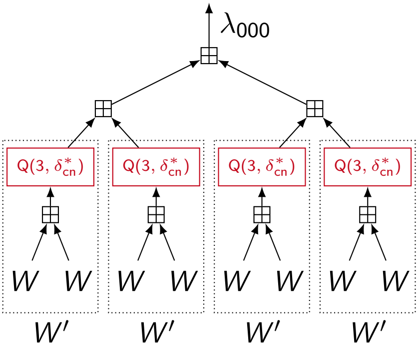

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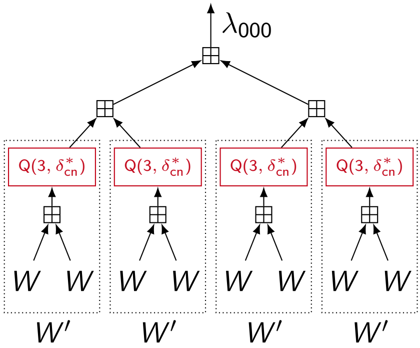
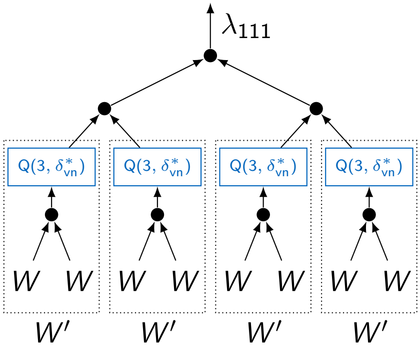


Variable nodes:



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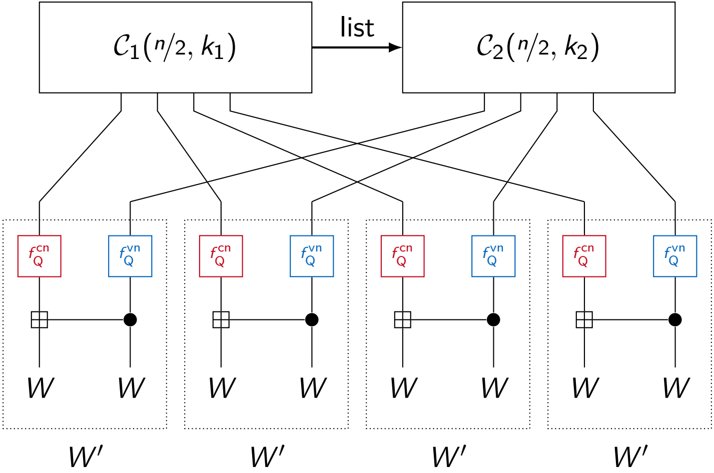

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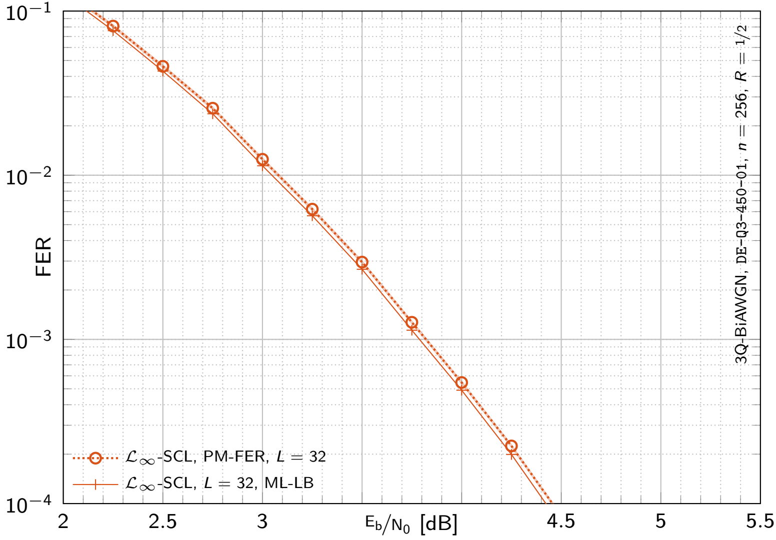

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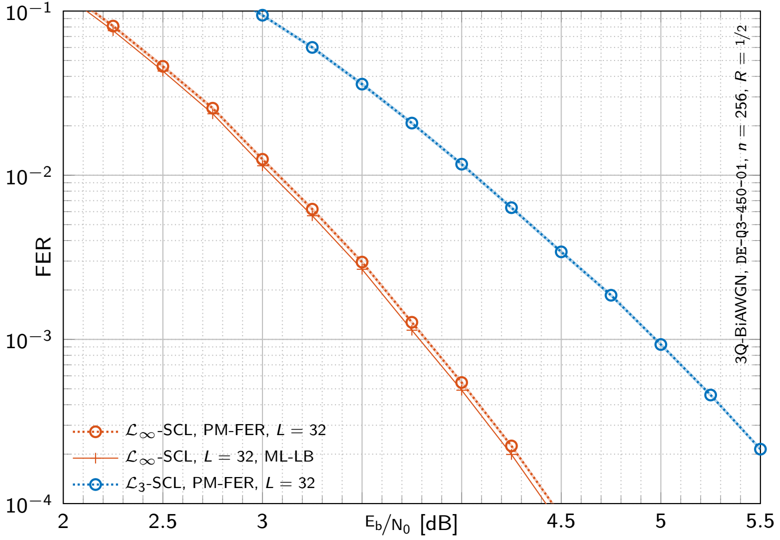

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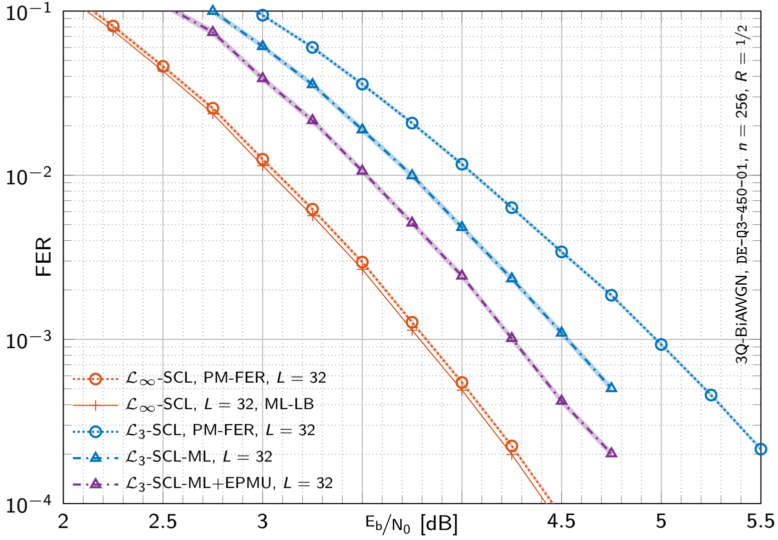

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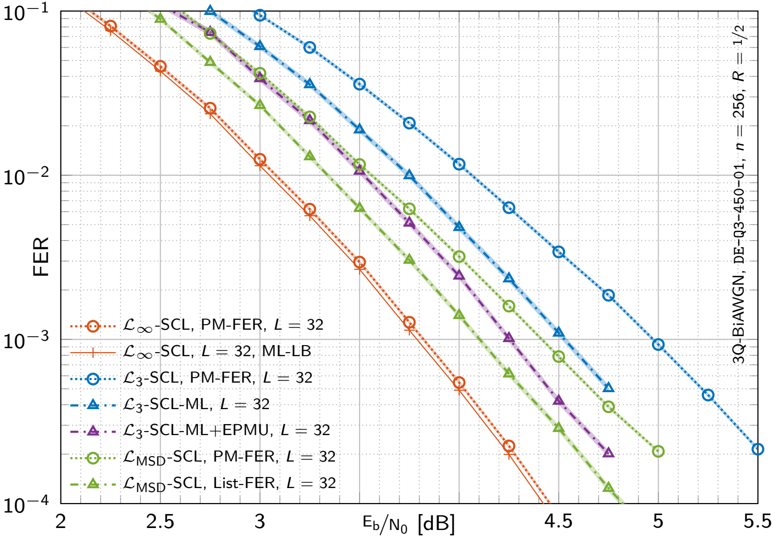

Conclusion

Key Take-Aways

- LLR quant. $\rightarrow$ PM quant. $\rightarrow$ list management impaired

$n = 256$, FER 10^{-3}

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- *ML-among-list* and *EPMU* can mitigate impact of quant.:

$n = 256$, FER 10^{-3}

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- Interactions with outer codes (*e.g.*, parity-checks)

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Thank you!

J. Neu, "Quantized Polar Code Decoders: Analysis and Design", Master's thesis at Technical University of Munich, September 2018, arXiv:1902.10395

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